Adding the fourth variable to the surface algebraic equation

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1. ADDING THE FORTH VARIABLE TO ALGEBRAIC SURFACE EQUATION

The algebraic surface in our case is the N-degree polynomial. We added one more variable to the equation. This variable represents a surface property: pressure in every point, temperature, density, etc. The value of the property in each point of the surface is visualized by means of color. Functional dependence between shape and a property (the fourth variable) can be different:

- linear depentanizer;
- squared relationship;
- transcendental;
- algebraic higher order equations.

An equation with four variables will be called color equation.

The first question appeared is connected solving the equation with four variables. How the equation solutions will be found if the number of the variables is bigger than the number of the equations in the equations set. There are two decisions:

- The property is a constant value for the whole surface. Fix the color. In this way the color will manage the shape. Changing the color we will change the surface.
- The property is irregularly spread on the surface. The surface will appear from solving the equation when we artificially set color to zero (to one). This will be our surface. Then we will find the color in every point of this surface by solving the equation with one variable (x, y and z are known now).

2. COLOR POLYNOMIAL

In the table below you can see the number of the coefficients of the color polynomial. The formula for calculating these numbers is represented below the table.

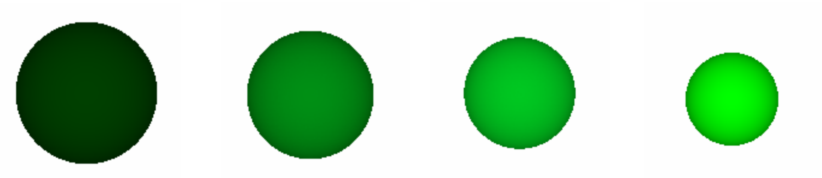
Power (k)	Number of the variables (n)		
	2	3	4
1	3	4	5
2	6	10	15
3	10	20	35
4	15	35	70
5	21	56	126

$$\{N_{k,n}\} = \{N_{k-1,n}\} + c^{k}_{n+k-1} = N_{i-1} + \frac{(n+k-1)!}{k!(n-1)!}$$

3. THE PROPERTY IS A CONSTANT VALUE FOR THE WHOLE SURFACE

In this chapter you can see the example of the first approach to the solving of the color equation. The example illustrates the relation between the property value and the geometry of the object.

Let’s take the n-degree polynomial with four variables, which gives us sphere surface: -10x – 10y – 10z = -400 + c.
The geometry equation : -10x – 10y – 10z = -400 (c is set to zero).
The color equation : -10x – 10y – 10z = -400 + c, where x, y and z are constants defining the surface point.
The visualization of this color equation is represented on Fig. 1. You can see that changing the property value changes the surface. You can imagine a ball being moved to the area with greater pressure.



а) c = 64G; б) c = 128G; в) c = 192G; г) c = 255G.
Fig. 1

4. THE PROPERTY IS IRREGULARLY SPREAD OVER THE SURFACE

4.1. Assigning roots of color equation to a color value

4.1.1. Roots absence

For example, some square polynomial may have no roots at all. Form disappears, because there is no color for a pixel. The way out is setting some color for no root case. The part of the surface painted with this color would give an idea of no root for color equation.

4.1.2. One or more roots

An equation may have more then one root. Several roots must have a single meaning – one color value. What should we do in this case? How can we unambiguously confront several roots with each other?

So, we’ll try to find a solution. Meaning of these relation “roots-color” is the secondary question for now. The point is to find different ways of visualization. Trying to find the sense of the formula is an application of it to the real world, is a connection with outward things. It is the implementation step of adding the forth variable to the algebraic surface equation.

- C_j - color of some surface point. It is the arithmetic mean of all color equation roots.

$$C_j = \frac{\sum_{i=1}^{NumbeOfRoots} C_i}{NumberOfRoots} \qquad \{1\}$$

where C_i – color equation root, $NumberOfRoots$ – number of roots.

- Using some more difficult formula, where input parameters are color equation roots and output parameter is color in this point.

Only the type of relation, where a point calculated color is the arithmetic mean of all color equation roots, is viewed here. Later on other formulas would be analyzed. The kernel of the approach is to analyze different ways of roots presentation. It was considered to be a game of getting interesting colorations. Some of these color schemes would virtually change the surface. For example, a sphere with a carving which is just a result coloration.

4.2. Analyzing free coefficient of the color polynomial

Analyzing the influence of the coefficient at the zero power unknowns it was found: in the case of detaching the form equation from the color equation by setting color variable to zero – any change of this coefficient would result in surface modification.

Color equation : $c^2 - 65025c - 10x^2 - 10y^2 - 10z^2 + 500 = 0$

Surface equation : $-10x^2 - 10y^2 - 10z^2 = -500$

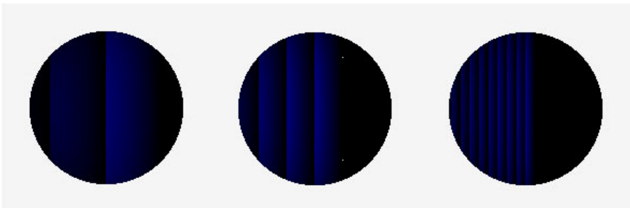
Let's change the free coefficient:

Color equation : $c^2 - 65025c - 10x^2 - 10y^2 - 10z^2 + 300 = 0$

Surface equation : $-10x^2 - 10y^2 - 10z^2 = -300$

As you can see the surface is changed by changing the free coefficient. The radius of the sphere is changed.

4.3. 2-order color polynomial visualization



a) b) c)

Fig. 2. Examples of changing coefficients at xc, yc, zc

a) $50xc + c^2 - 255c - 10x^2 - 10y^2 - 10z^2 + 500 = 0$;

b) $100xc + c^2 - 255c - 10x^2 - 10y^2 - 10z^2 + 500 = 0$;

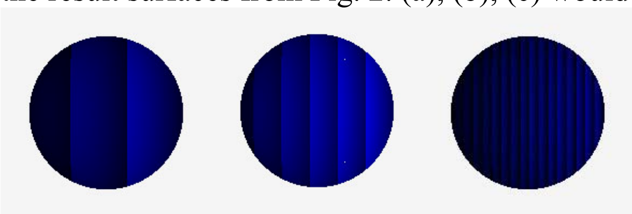
c) $300xc + c^2 - 255c - 10x^2 - 10y^2 - 10z^2 + 500 = 0$.

In these visualizations the range of possible calculated colors (see formula {1}) coincide with color range from 0 (00 00 00 - black) till 16777215 (FF FF FF - white). Values calculated from color equation and formula {1}, which appeared to be out of range, are set to closest border value. Formulas and their visualizations (a), (b), (c) represent the examples of discrepancy of calculated and visual ranges. The range of calculated colors for (b) is [-445, 955] but the range of real color values is [0, 16777215]. As the result right half of the sphere is black: bottom real color limit. The way out would be shifting the calculated color values in order to fit the real color range:

$$CC_j = C_j - Min(C_j), \qquad \{2\}$$

where C_j – color of some surface point calculated using formula {1},
$Min(C_j)$ – minimum calculated color value of the seen surface.

So first the matrix of projection solutions of color equation (several roots are assigned to some color value) is formed, then using the border correction the final color of a point is calculated. As the result surfaces from Fig. 2. (a), (b), (c) would be visualized like (a), (b), (c) from Fig. 3.



a) b) c)

Fig. 3. Coloration with range correction

So to the algorithm of setting relation "color equation roots - color" one more point should be added: shifting values in order to fit real color range and to avoid out of limit values. Or on another hand a premeditated inconsistency between ranges may be added to highlight some important values.

One more example is represented on Fig. 4

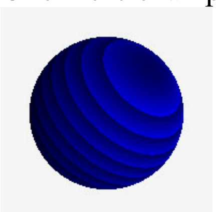


Fig. 4.

$$100xc + 100yc + 100zc + c^2 - 255c - 10x^2 - 10y^2 - 10z^2 + 500 = 0,$$

$$CC_j = C_j - Min(C_j), \text{ where } C_j = \frac{\sum_{i=1}^{NumberOfRoots} C_i}{NumberOfRoots}, \quad Min(C_j) = -969.$$

4.4. Color maps

Color map is used to assign a property value to a color. For example, if a property value of the surface point is bigger them 0 but lower than 100 then the point is painted with red, [101; 200] - green, [201; 300] - blue. This is the simplest color map. Color maps allow to get more difficult colorations.

Color map used for the following example (Fig. 5) is:

$(MeanRoot < -2)$): color = 00007F,
$(MeanRoot \geq -2)$ and $(MeanRoot < -0.5)$: color = 00000FF,
$(MeanRoot \geq -0.5)$ and $(MeanRoot < 0)$: color = 007F80,

$(MeanRoot) \geq 0$ and $(MeanRoot < 0.5)$: color = 00FF00,
$(MeanRoot \geq 0.5)$ and $(MeanRoot < 2)$: color = 100000,
$(MeanRoot \geq 2)$: color = FF0000.

$MeanRoot$ is a mean value of color equation roots in each surface point.

$MeanRoot = C_j - Min(C_j)$, where $C_j = \dfrac{\sum_{i=1}^{NumberOfRoots} C_i}{NumberOfRoots}$

As I already said, some other formula may be used for getting MeanRoot from several color equation roots. Another formula will give different visualization of the property values. The selected formula may be based on the answer on the question what is worth seeing: critical values, or roots absence or etc. Thus, for example, the ranges of the critical values may be highlighted with some bright colors.

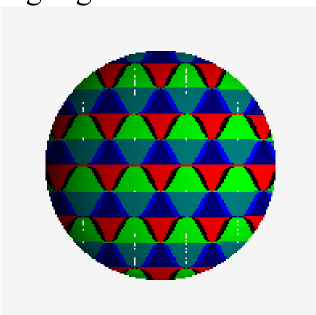
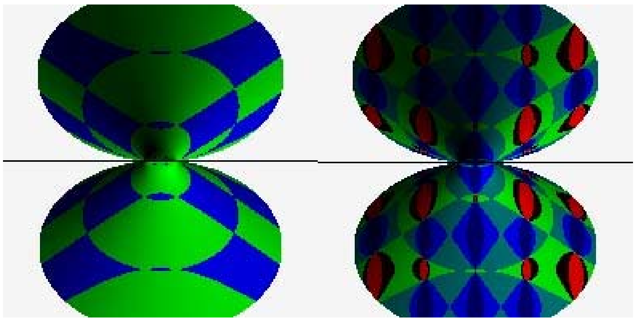


Fig. 5

$100Sin(x)Sin(x)Sin(z)c^2 + 100Cos(y)Cos(y)Cos(z)c - 10x^2 - 10y^2 - 10z^2 + 500 = 0$



a) б)

Fig. 6

a) $100Sin(x)Sin(x)c^2 + 100Cos(y)Cos(z)c - 10x^2 - 10y^2 + 10z^2 + 10 = 0$;

Color map:

$(MeanRoot = 0)$: color = Red,
$(MeanRoot < 0)$: color = Green,
$(MeanRoot > 0)$: color = Blue.

б) $100Sin(x)Sin(x)c^2 + 100Cos(y)Cos(z)c - 10x^2 - 10y^2 + 10z^2 + 10 = 0$;

Color map:

$(MeanRoot < -2)$): color = 00007F,
$(MeanRoot \geq -2)$ and $(MeanRoot < -0.5)$: color = 00000FF,
$(MeanRoot \geq -0.5)$ and $(MeanRoot < 0)$: color = 007F80,

$(MeanRoot) \geq 0$ and $(MeanRoot < 0.5)$: color = 00FF00,
$(MeanRoot \geq 0.5)$ and $(MeanRoot < 2)$: color = 100000,
$(MeanRoot \geq 2)$: color = FF0000.

You can see that the color equation is the same for both images on Fig. 6, but the coloration is different. It was achieved by using different color maps. The color map for the first image on Fig. 6 is simple. As the result we got two-color surface. The second color map is more difficult that results in the more difficult colorization.

5. CONCLUSION

Adding the fourth variable to the algebraic surface polynomial is a new approach to the surface coloring. First major steps are done already. Solving equations sets with the number of variables more than the number of equations are analyzed, described and the ways around are found. Also we got some results of dealing with several roots assignment to one color, the list of color schemes would be added. Getting more color schemes, using some other formulas for relation "Color equation roots-color", analyzing the higher order color equations are in sight. More difficult color formulas should give come complicated color schemes.