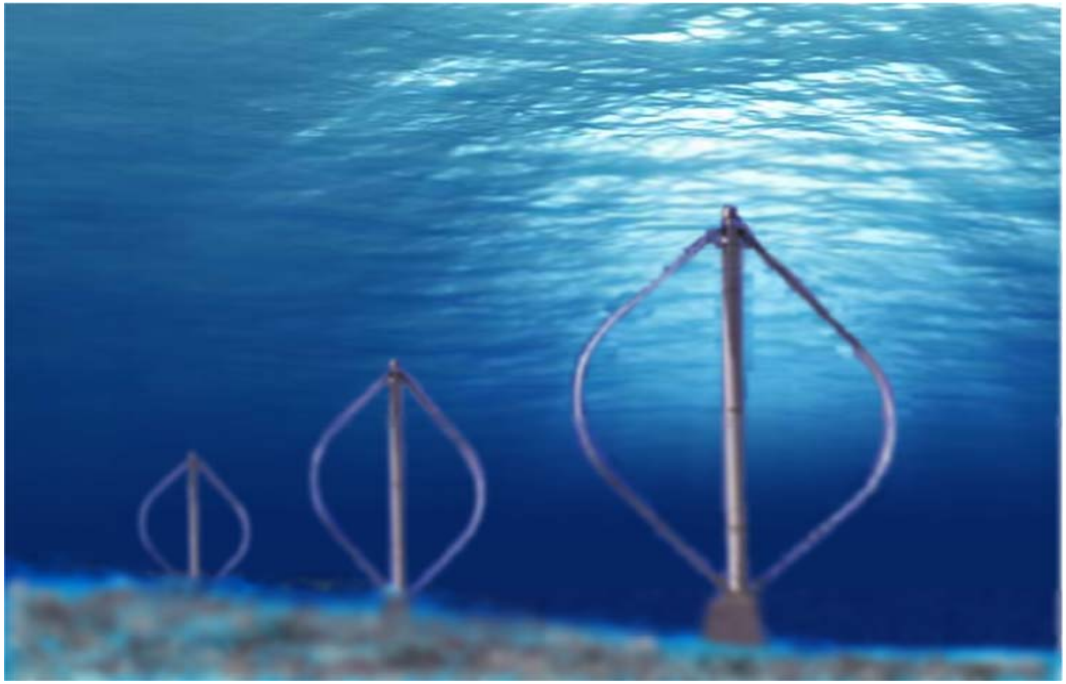
# *Investigations on current mills and wind turbines*



**Theodor Beisheim**

***JASS 2008***

**St. Petersburg**

**March 9th -19th 2008**

# *Investigations on current mills and wind turbines*

- Introduction
- Boundary Conditions
  - wind vs. tidal currents
- Theory of energy conversion
  - Betz' theory
  - airfoil theory – lift and drag
- *vertical axis free stream turbines*
  - vertical vs. horizontal axis
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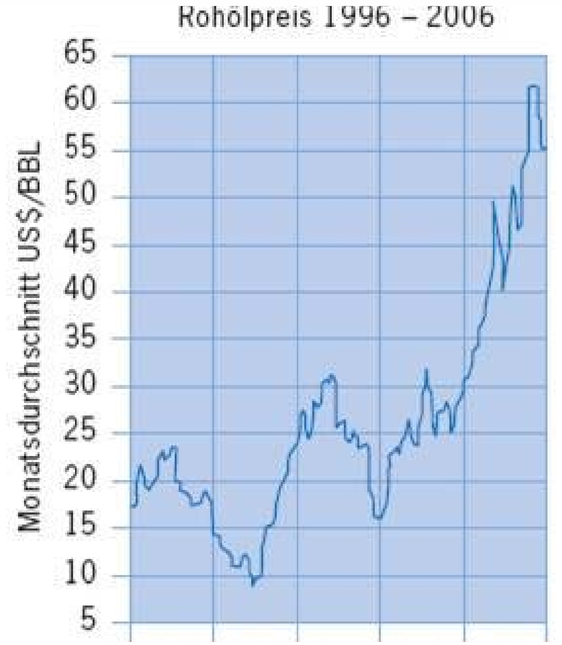
  - computational methods for drag based design at low TSR

# Introduction

•The energy situation is getting more difficult




• the will to reduce greengas emissions

# Introduction

possible alternatives:

- e.g. nuclear power
- renewable energy resources

biomass

hydro

solar

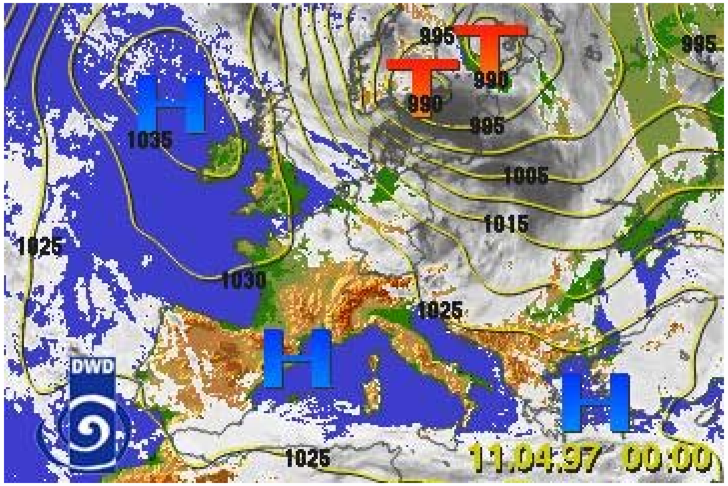
wind

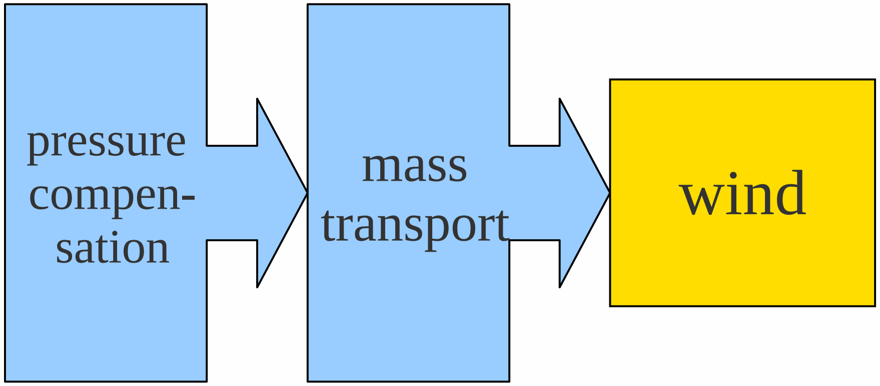
tidal currents

# Introduction

**wind generation:**

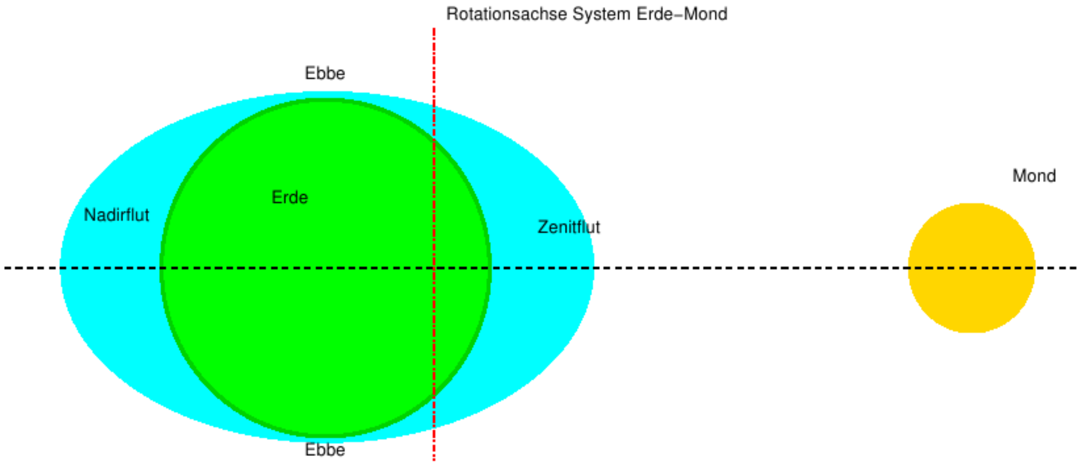
- due to geographical diffenrences, there are developing areas of our atmosphere of higher and some of lower pressure

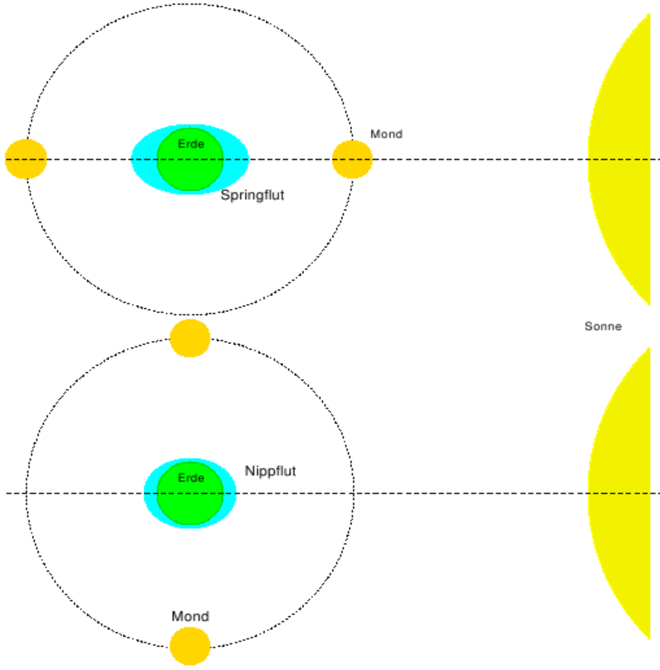


# Introduction

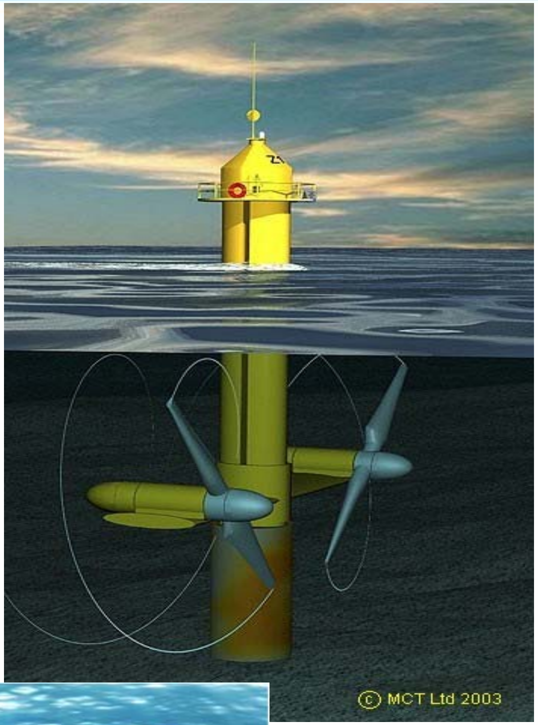
**development of tidal currents: - cyclic changes in the sea level**
**- due to gravitation forces in the interplanetary system Sun – Moon - Earth**

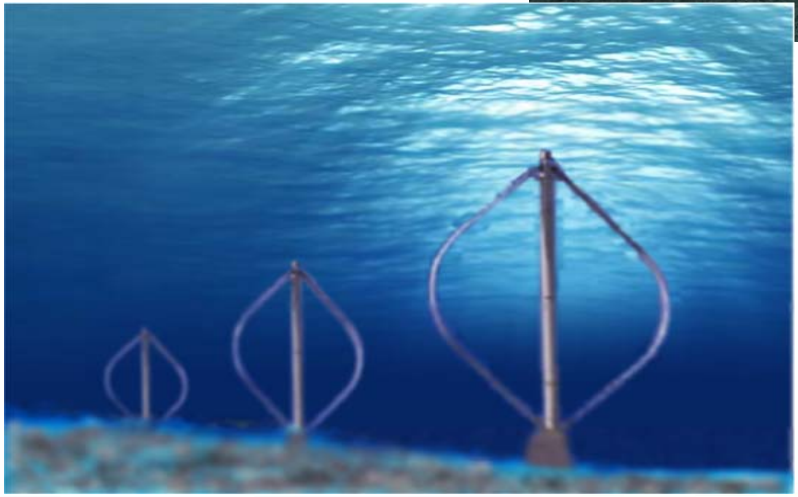


# Introduction

examples:



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# boundary conditions

## wind vs. tidal currents:

| | wind | tidal currents |
|---|---|---|
| density ρ | 1,2 kg/m³ | 1000-1020 kg/m³ |
| viskosity η | 0,0000171 N/(m² s) | 0,001 N/(m² s) |
| velocity v | 10 m/s | 2 – 3 m/s |
| Re | **300000** | **450000** |

$$Re=\rho \cdot v \cdot L \cdot \eta^{-1}$$

# Boundary conditions

**wind vs tidal currents**

**- same Re – numbers in wind and water:**

**we can use technologies proved in wind energy to convert energy of tidal streams**

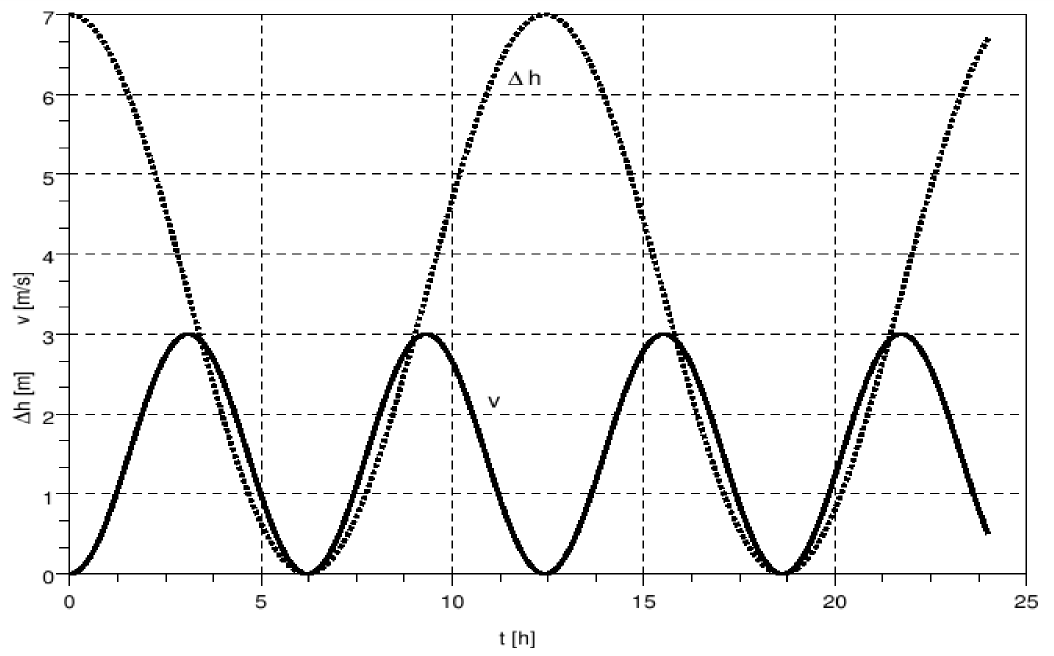
**- water and wind streams through free stream turbines can both be calculated incompressible**

?

# Boundary conditions

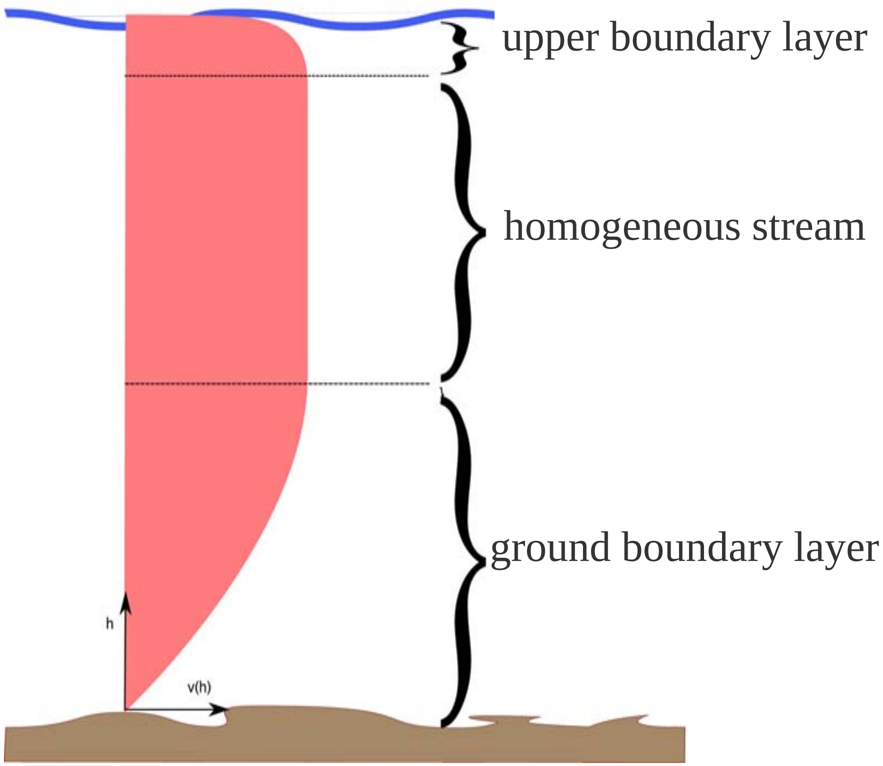
**major advantage of tidal currents in comparison to wind streams:**

- cyclic variation of sealevel and the velocity of the stream are **predictable**



# Boundary conditions

**boundary layer:**




-influences of surface roughness to the velocity profile

- vertical energy transfer leads to turbolence

- at the ground : $v(z=z_0)=0$

- second boundary layer water-air !

# Boundary conditions

conclusions: - direction of the stream changes twice a day
- machine should not be sensitive to see level change / vertical velocity change

possible solution: -*vertical axis free stream turbine*
- converts power from every stream direction
- this compensates the lower efficiency
- generator and transmission could be built over the water surface

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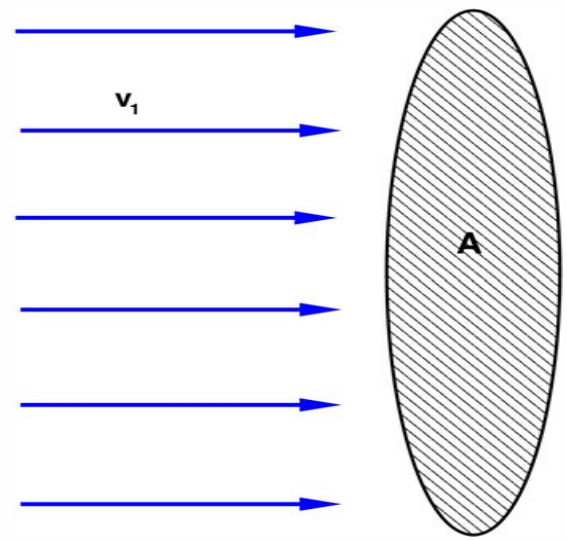
# theory of energy conversion

**Betz' theory:**

**total wind power through a control area A :**

$$P_{wind} = \frac{dE_{wind}}{dt} = \frac{d}{dt}(\frac{1}{2}mv_{\infty}^2) = \frac{1}{2}\rho A v_{\infty}^3$$

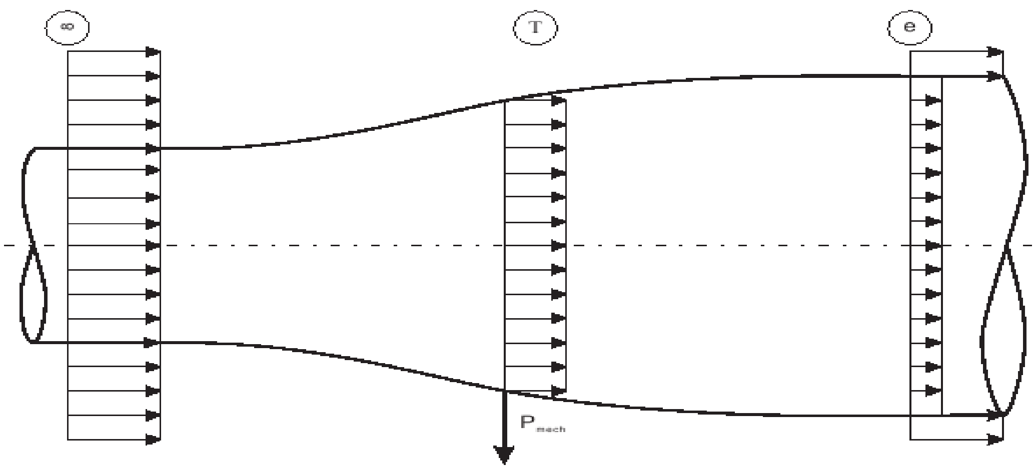



Equation of Continuity:

$$\rho v_1 A_1 = \rho v_2 A_2 = \rho v_3 A_3$$

Energy balance:

$$\Delta E = E_1 - E_3 = \frac{1}{2}m(v_1^2 - V_3^2)$$



# theory of energy conversion

**Betz' theory:**

Power extracted in the Turbine area: $P_{turbine} = \dot{E} = \frac{1}{2}\dot{m}(v_1^2 - V_3^2)$

mass flow rate through the turbine: $\dot{m} = \rho v_2 A_2$

with $v_2 = \dfrac{v_1 + v_3}{2}$ Froude- Rankine- theorem

Power extracted by the turbine: $\dot{E} = \frac{1}{2}\rho \frac{v_1 + v_3}{2} A_2 (v_1 + v_3)(v_1 - v_3)$

in relation to the windenergy:

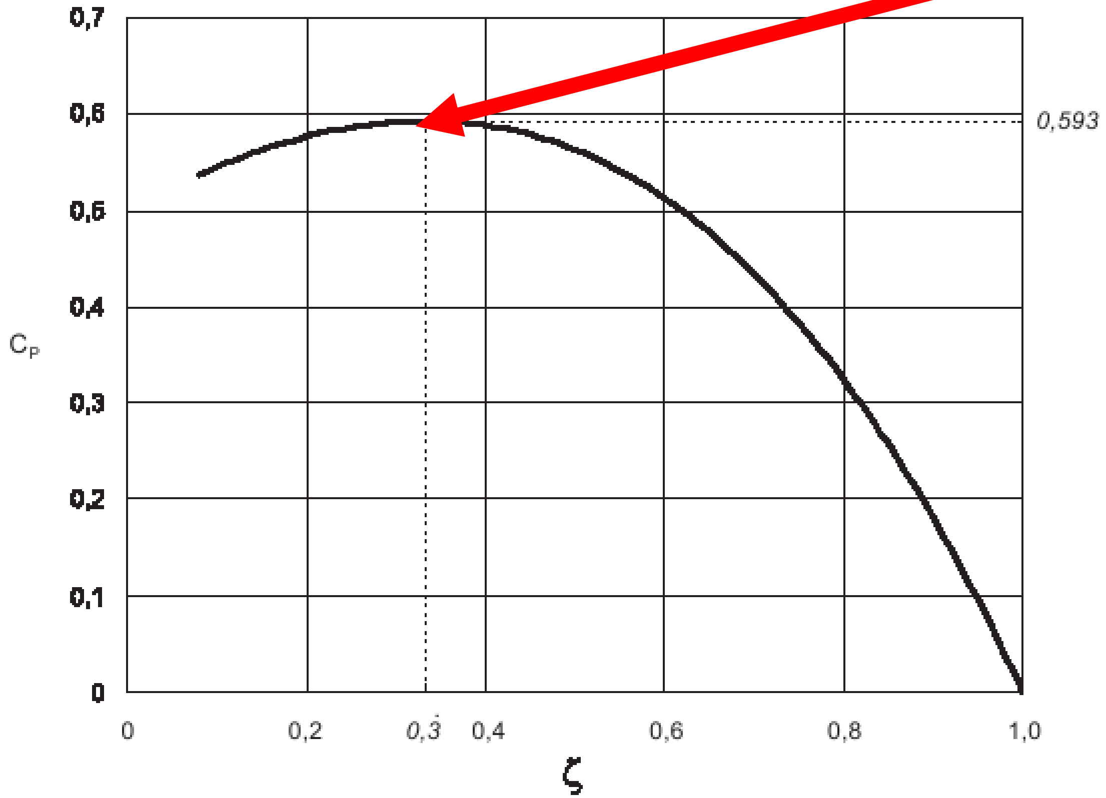
$$\dot{E} = \underbrace{\frac{1}{2}\rho A_2 v_1^3}_{P_{wind}} \underbrace{\frac{1}{2}\left(1 + \frac{v_3}{v_1} - \frac{v_3^2}{v_1^2} - \frac{v_3^3}{v_1^3}\right)}_{c_P}$$

# theory of energy conversion

## Betz' theory:

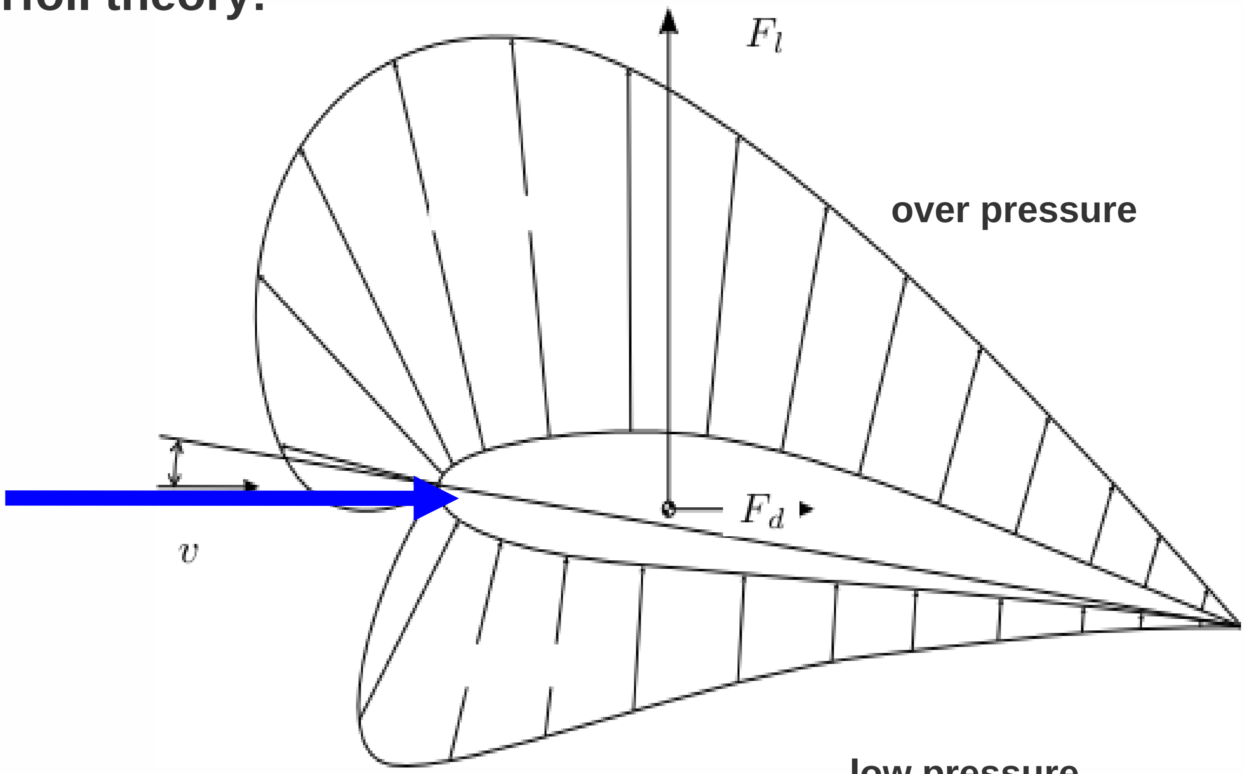
maximum efficiency:

$$c_{P_{Betz}} = \frac{16}{27} \approx 0,59$$



# theory of energy conversion

**airfoil theory:**



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- ***vertical axis free stream turbines***
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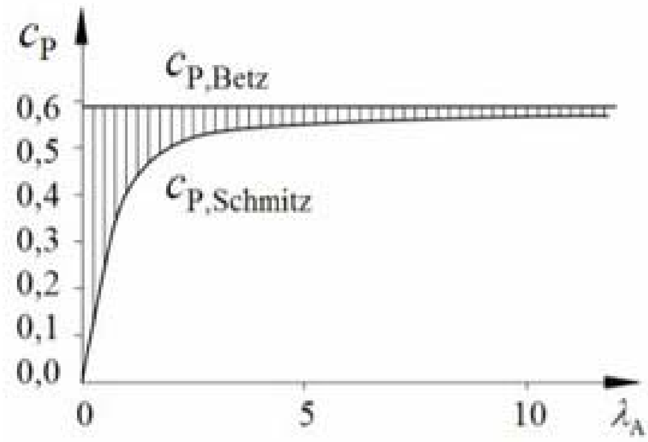
# vertical axis free stream turbine

## vertical vs. horizontal axis:

horizontal axis machines:

-significant losses due to angular momentum at low velocities

Schmitz' Theor




- sensitive to sea level change

rotation in 2 phases

- damages the blade
- severe decrease of power

# vertical axis free stream turbine

## vertical vs. horizontal axis:

vertical axis machines:
- not sensitive to sea level changes
- Transmission and Generator can be built above sea level
- efficient with low velocities

Investigation of vertical axis turbines for energy conversion from tidal currents!

lift based designs

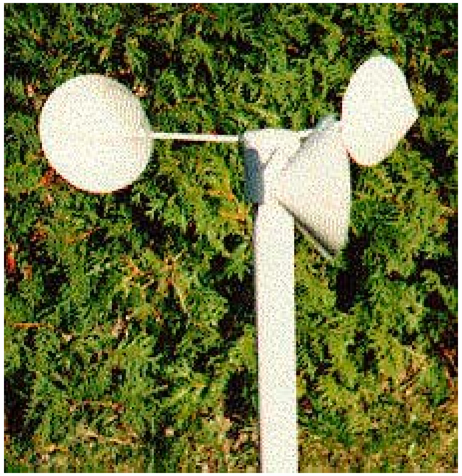
drag based designs

# vertical axis free stream turbine

**drag based turbines:**

-oldest and simplest way to extract energy from the wind

anemometer



persian wind turbine

# vertical axis free stream turbine

**drag based turbines:**

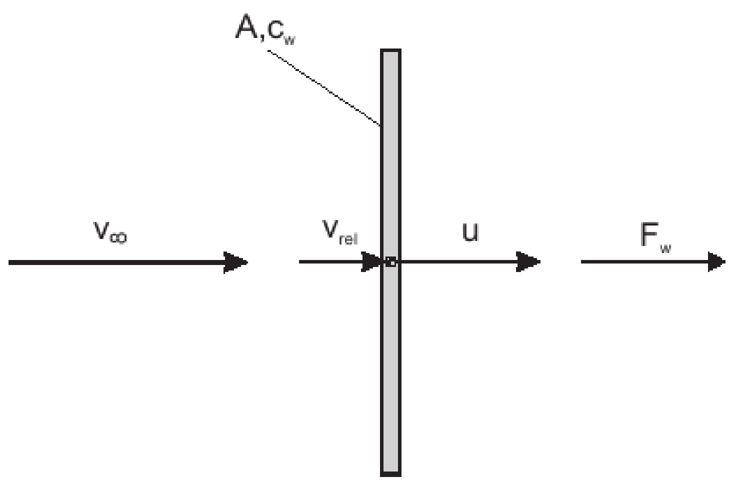
**theory:** - area A with moving with velocity u

- wind speed v

Power: $P_T = F_W u$

Force: $F_W = \frac{\rho}{2} c_W A v_{rel}^2$

with $v_{rel} = v_\infty - u$

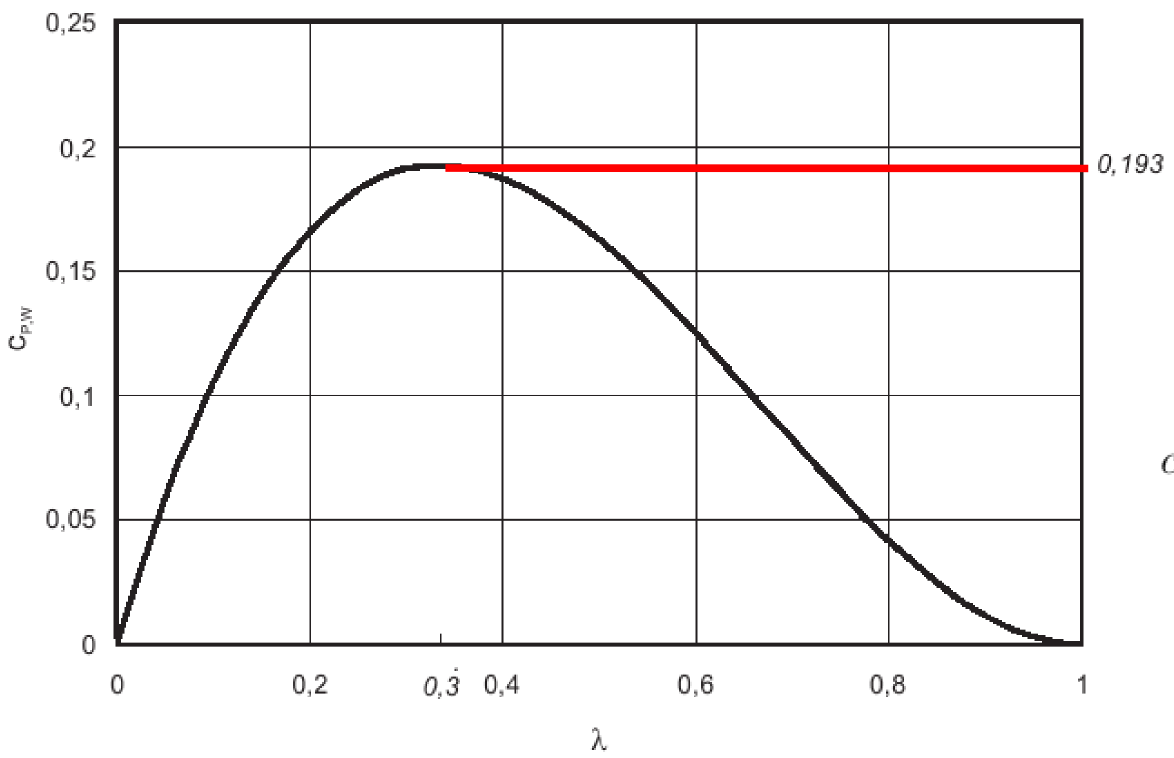



$$P_T = \frac{\rho}{2} c_W A (v_\infty - u)^2 u$$

# vertical axis free stream turbine

**drag based turbines:**

power coefficient: $$c_{P,W} = \frac{c_W (v_\infty - u)^2 u}{v_\infty^3} = c_W \left(1 - \frac{u}{v_\infty}\right)^2 \frac{u}{v_\infty}$$




with $$\lambda = \frac{u}{v_\infty} = \frac{r\omega}{v_\infty}$$

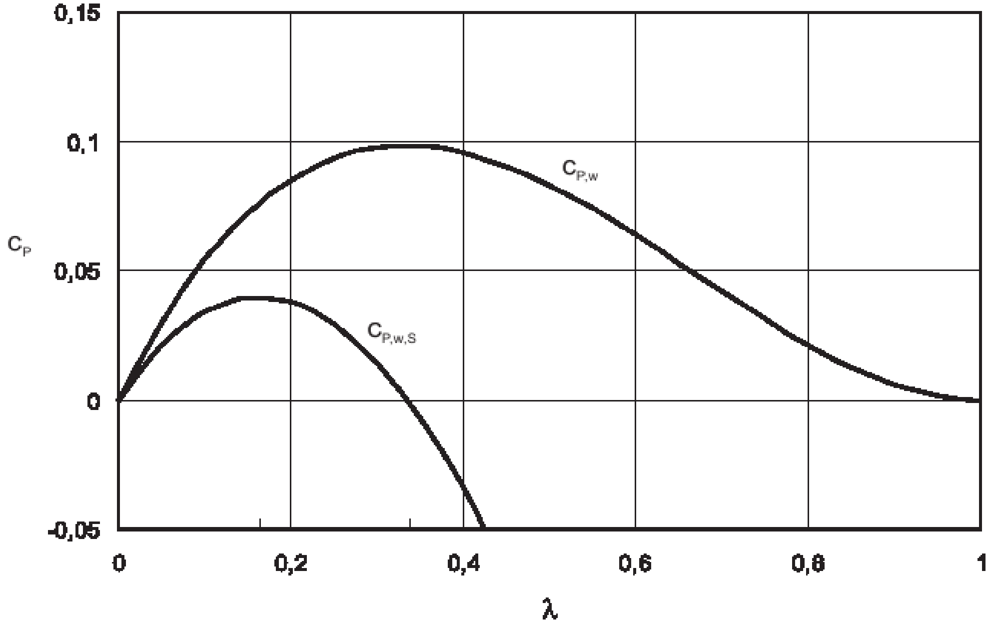
$$c_{P,w} = c_w \frac{A}{A_{Rotor}} \left(1 - \lambda\right)^2 \lambda$$

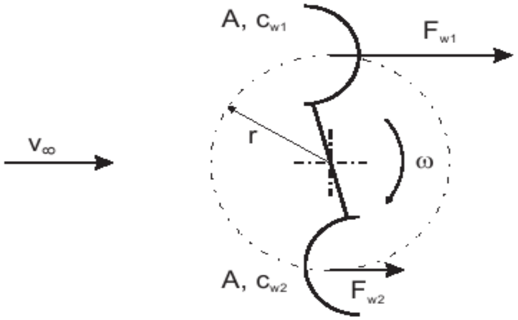
# vertical axis free stream turbine

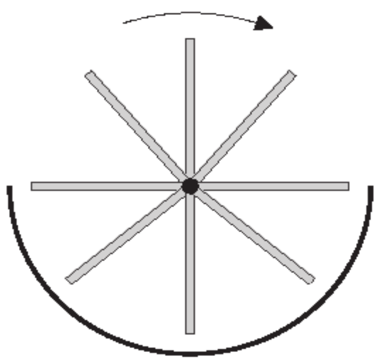
drag based turbines :

-drag forces on the way back

**real power coefficient:**



# vertical axis free stream turbine

## Lift based turbines:

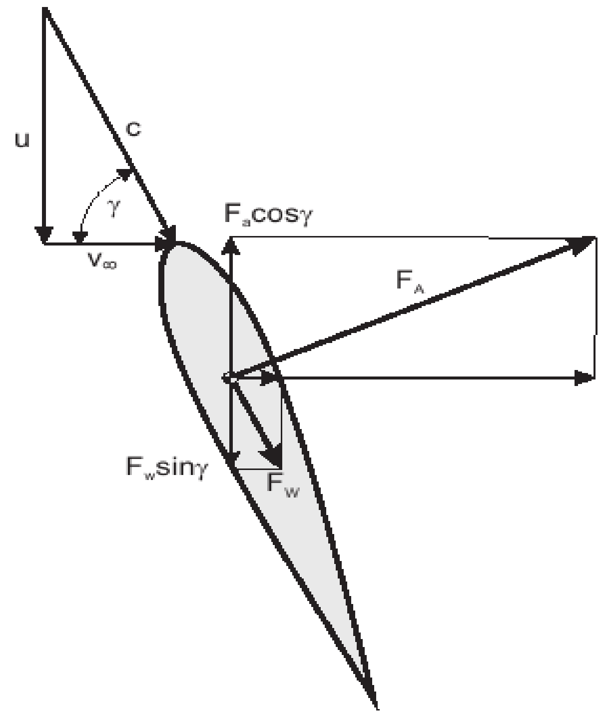
Darrieus turbine

H-Darrieus turbine

***How can we calculate the power of such a machine?***

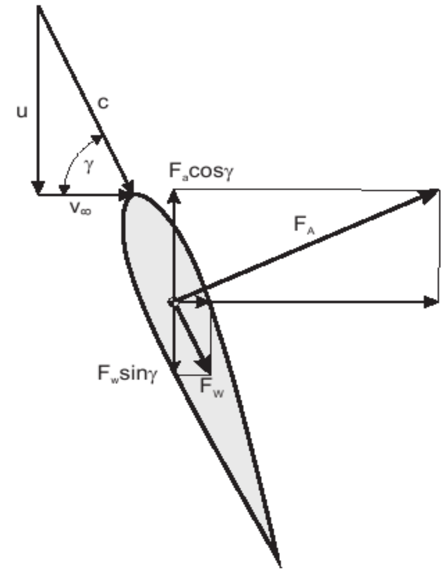
# vertical axis free stream turbine

**lift based design:**



# vertical axis free stream turbine

**lift based design:** theory:




velocity:

$$\tan\gamma = \frac{u}{v_\infty} = \lambda$$

power:

$$P = (F_A \cos\gamma - F_W \sin\gamma)u$$

$$P = \frac{\rho}{2} A v_\infty^3 c_A \lambda \sqrt{1+\lambda^2}\left(1 - \frac{\lambda c_W}{c_A}\right)$$

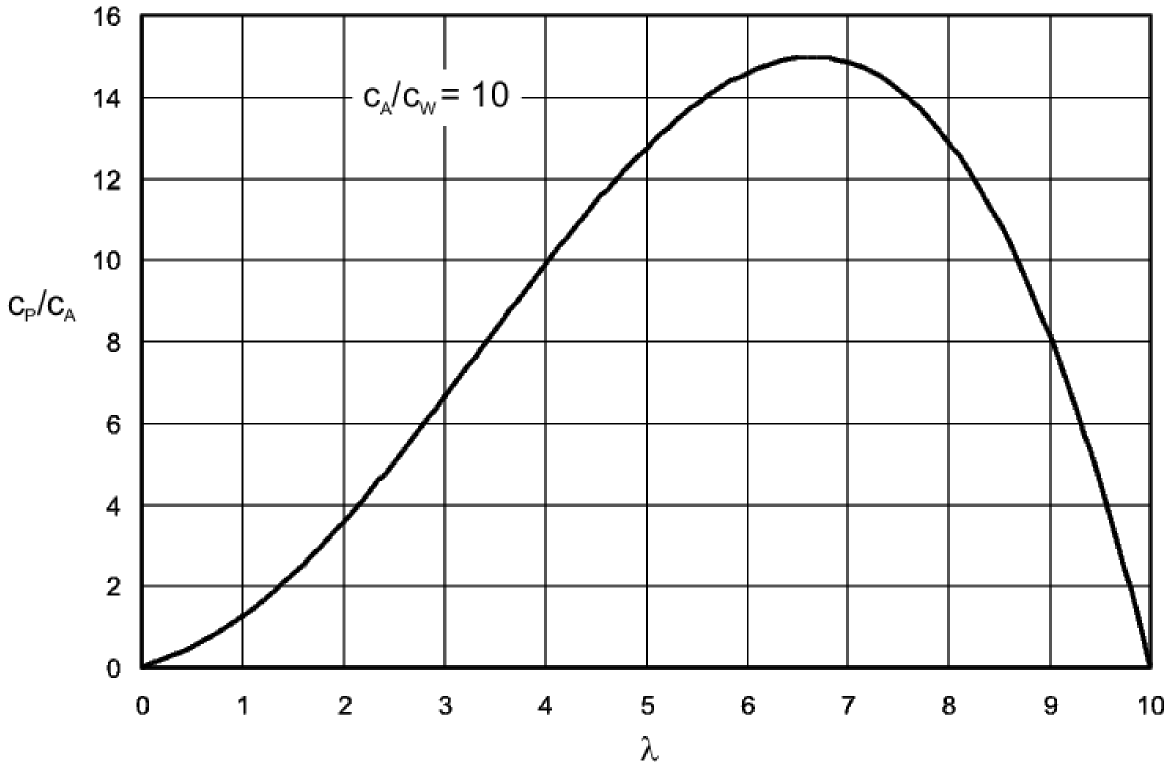
power coefficient:

$$c_{P,A} = c_A \lambda \sqrt{1+\lambda^2}\left(1 - \frac{\lambda}{\frac{c_A}{c_W}}\right)$$

# vertical axis free stream turbine

**lift based design:** theory:

$$c_{P,A} = c_A \lambda \sqrt{1+\lambda^2}\left(1 - \frac{\lambda}{\frac{c_A}{c_W}}\right)$$




ide ratio :

$$\epsilon = \frac{c_A}{c_W}$$

# vertical axis free stream turbine

**lift based design:** comparison to drag based design:

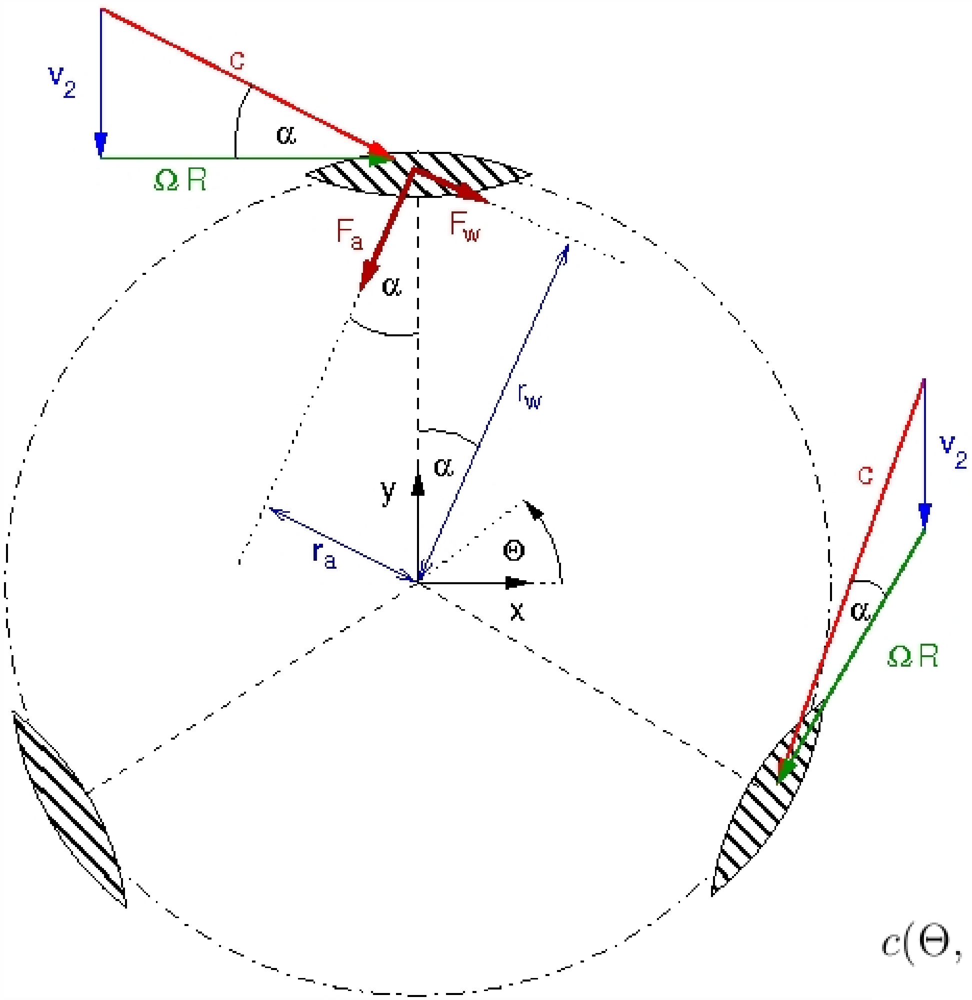
- ratio of the maximum power:

$$\frac{P_{max,A}}{P_{max,W}} \sim \left(\frac{c_A}{c_W}\right)^2$$

example: plate with glide ratio 10

use of lift is 100 times better than use of drag!

# vertical axis free stream turbine




**Lift based turbines:**

***main characteristics:***

**tip speed ratio:**

$$\lambda = \frac{\Omega\ R}{v_1}$$

**angle of attack:**

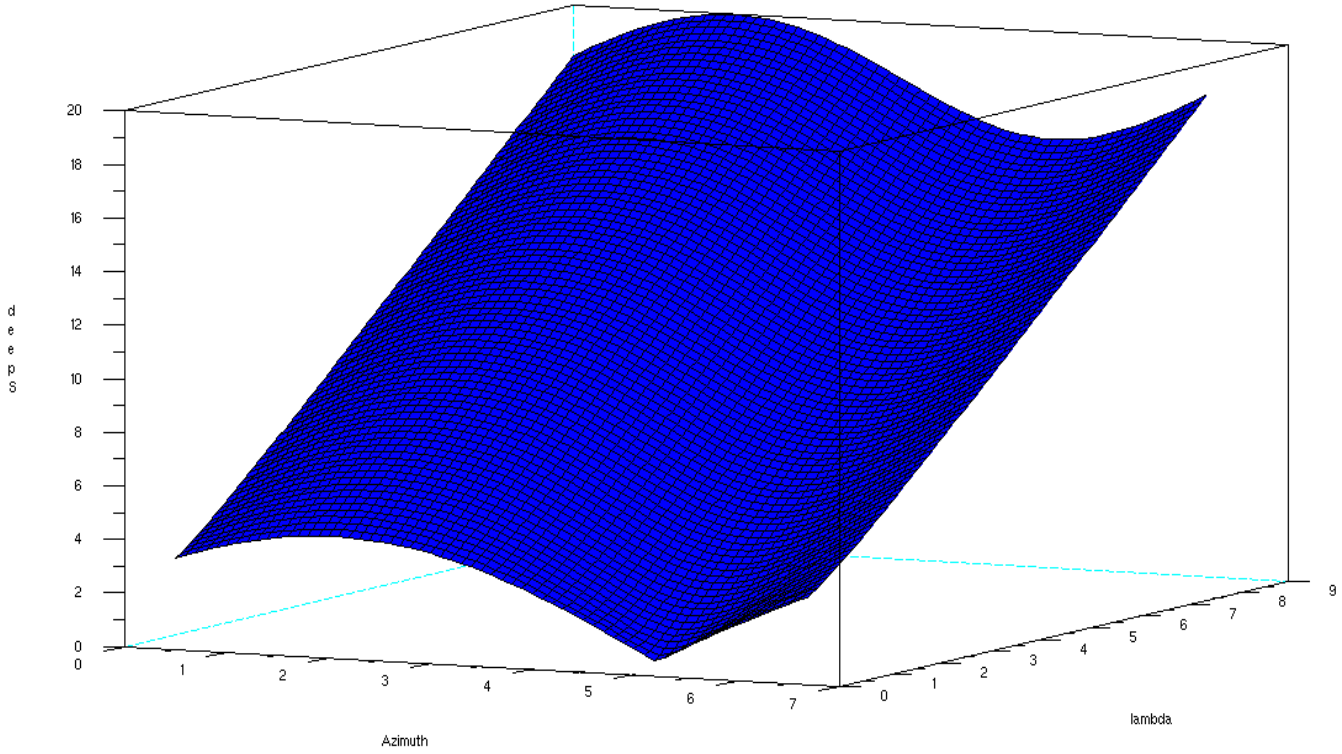
$$\alpha(\Theta,\lambda) = \begin{cases} \arcsin\left(\frac{cos\Theta}{\lambda}\right) & ,\frac{\pi}{2} < \Theta < \frac{3\pi}{2} \\ -\arcsin\left(\frac{cos\Theta}{\lambda}\right) & ,sonst \end{cases}$$

**relative velocity:**

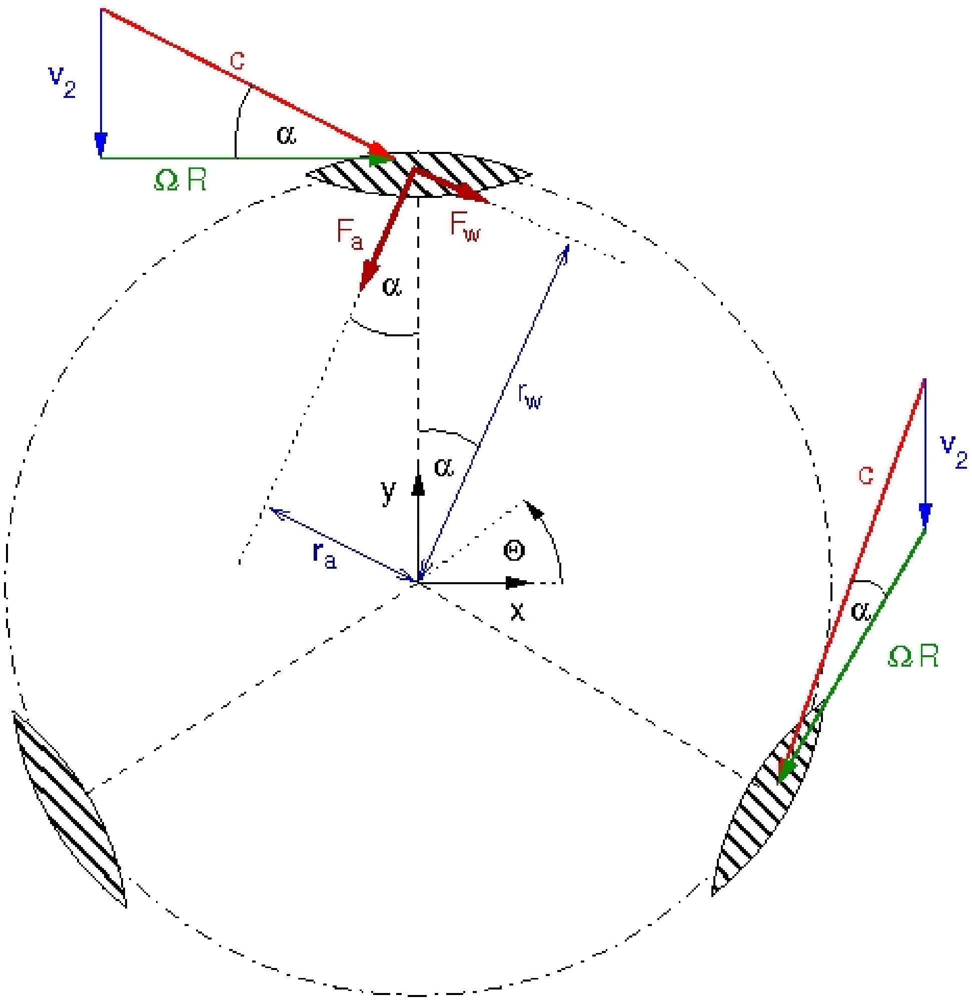
$$c(\Theta,\lambda) = \sqrt{(v_2 + \lambda \cdot v_2 \cdot sin\Theta)^2 + (\lambda \cdot v_2 \cdot cos\Theta)^2)}$$

# vertical axis free stream turbine

## Lift based turbines:



# vertical axis free stream turbine




**Lift based turbines:**

***Blade-Forces:***

**Lift:**

$$F_a(\Theta,\lambda) = c_a(\Theta,\lambda) \cdot \frac{\rho}{2} \cdot c(\Theta,\lambda)^2 \cdot t_P \cdot H$$

?

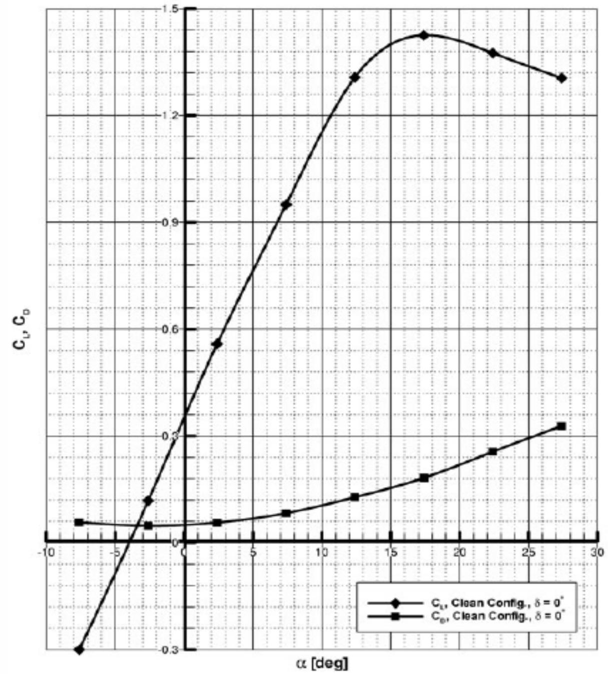
**Drag:**

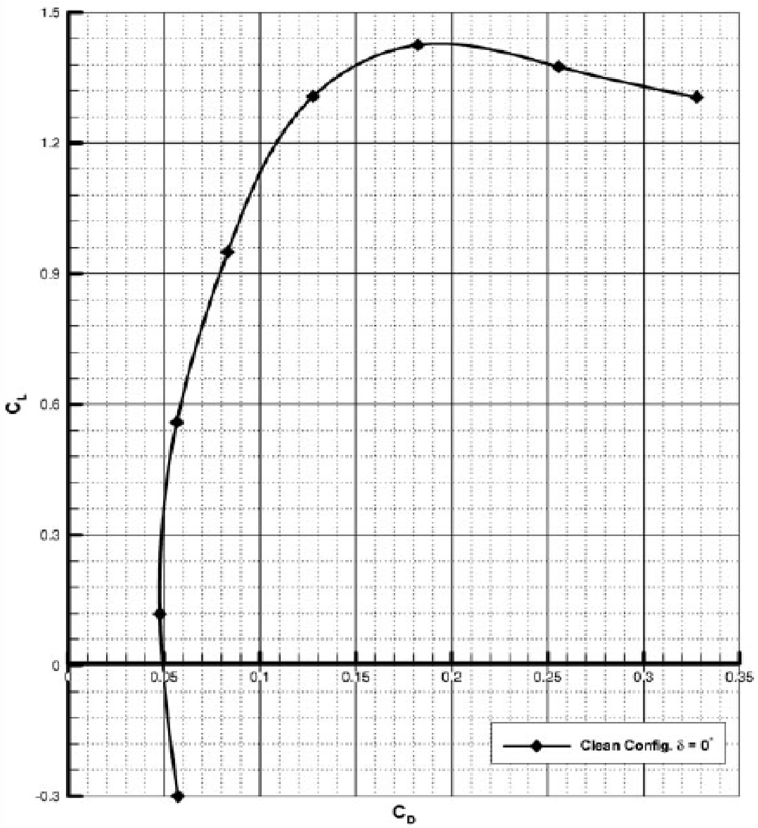
$$F_w(\Theta,\lambda) = c_w(\Theta,\lambda) \cdot \frac{\rho}{2} \cdot c(\Theta,\lambda)^2 \cdot t_P \cdot H$$

# vertical axis free stream turbine

**Lift based turbines:**

Lift and drag coefficients:




approximation:

$$c_a(\Theta, \lambda) = 2 \cdot \pi \cdot \sin(\alpha(\Theta, \lambda))$$

$$c_w(\Theta, \lambda) = \frac{c_a(\Theta, \lambda)}{100} + 0.04$$

# vertical axis free stream turbine

**Lift based turbines:**

Our goal: - power calculation of the turbine:

$$P_{Rotor}(\lambda) = n\ M_{Rotor,ges}(\lambda)\ \Omega$$

- torque produced by each blade:

$$M_{Rotor}(\Theta,\lambda) = F_a(\Theta,\lambda)\ r_a - F_w(\Theta,\lambda)\ r_w$$

arms of the lever

$$r_a = R\ \sin(\alpha(\Theta,\lambda))$$

$$r_w = R\ \cos(\alpha(\Theta,\lambda))$$

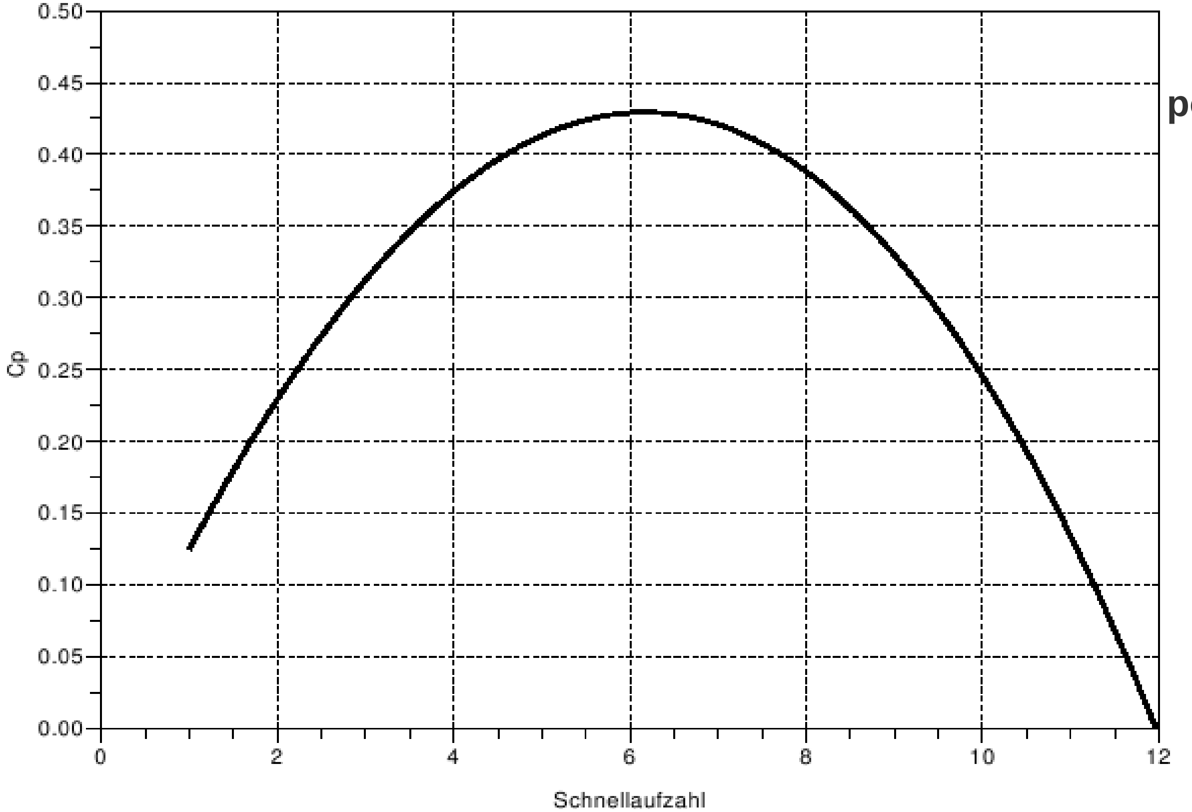
$$M_{Rotor,ges}(\lambda) = \int_0^{2\pi} M_{Rotor}(\Theta,\lambda)\ d\Theta$$

# vertical axis free stream turbine

**lift based turbine:**

**obtain the power**

$$P_{Rotor}(\lambda) = n\ M_{Rotor,ges}(\lambda)\ \Omega$$




**power coeffient:**

$$c_p(\lambda) = \frac{P_{Rotor}}{\frac{\rho}{2}\ v_1^3\ A_{Rotor}}$$

# vertical axis free stream turbine

**lift based design:**

**assumptions made:**

**-homogeneous stream through the turbine**

**- same velocity at all blades in every position**

**-angular momentum is to be neglected**

**validation of the method:**

**-influences of airfoil wake and angular momentum are essiential**

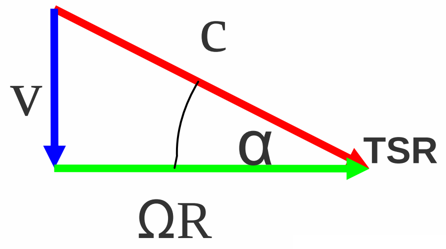
**- no possible at low tip speed ratio!**

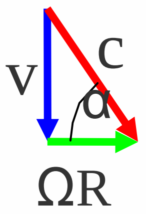
**this modell just for general lay out ( order of magnitude)**

# vertical axis free stream turbine

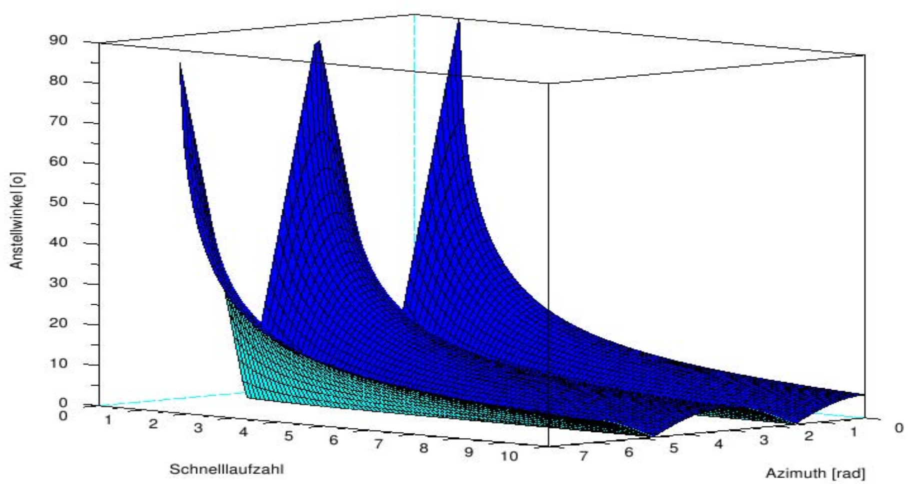
**lift based design: <u>behaviour at low TSR:</u>**

**<u>high TSR: low TSR</u>**



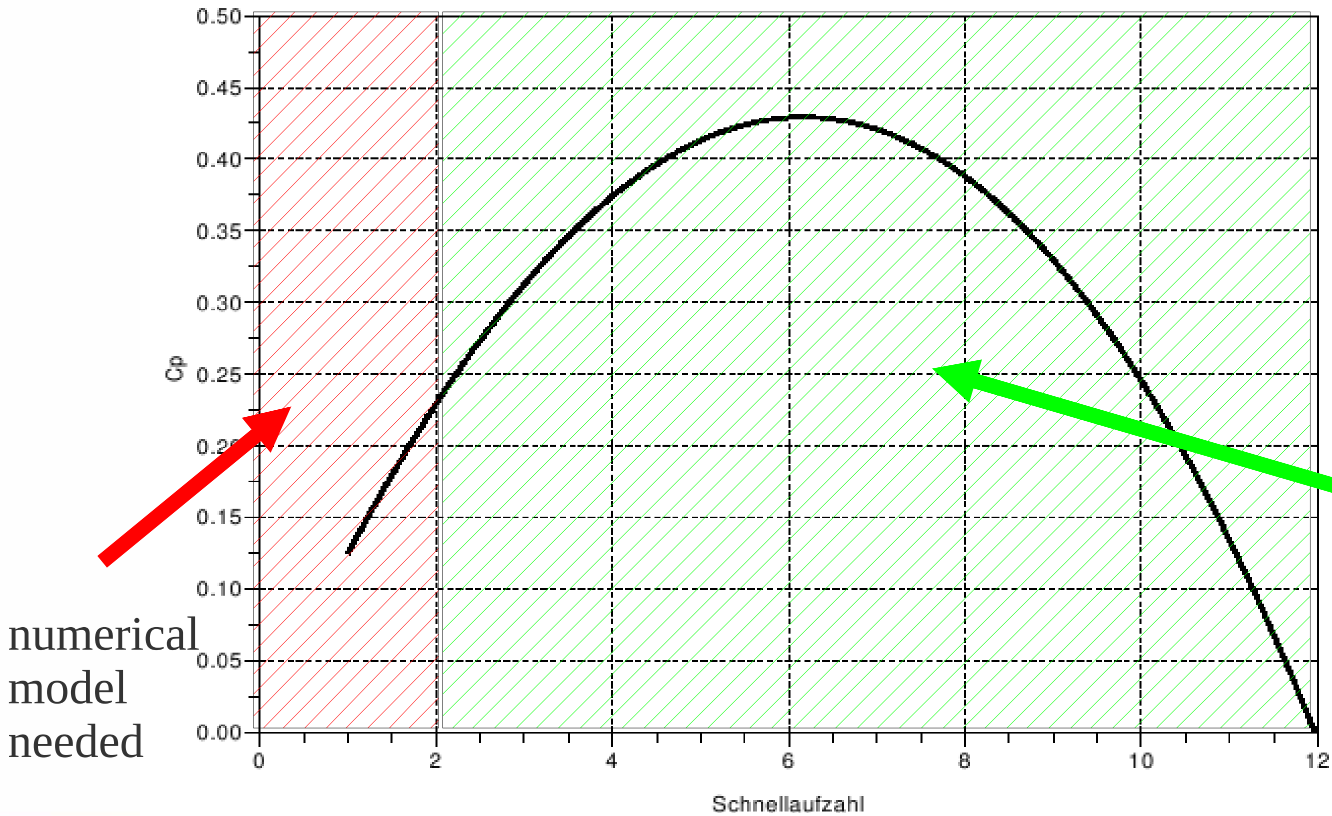




**- angular of attack rises with decreasing**



# vertical axis free stream turbine

**lift based design:** **behaviour at low TSR:**



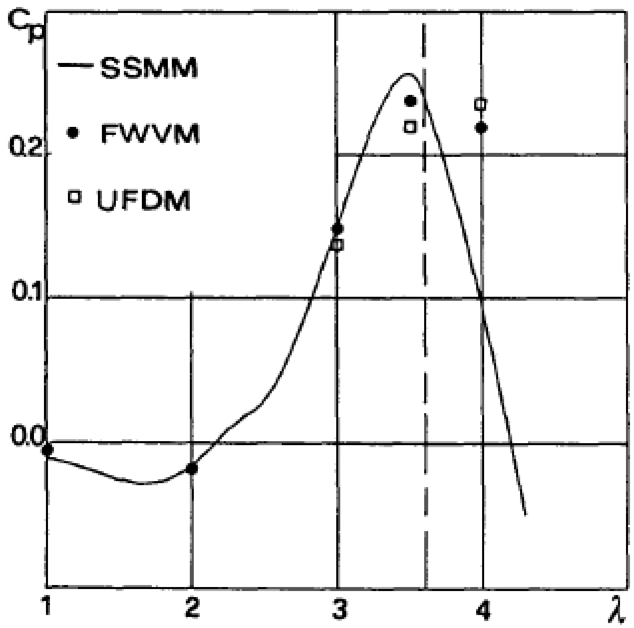

Power coefficient

$$c_p(\lambda) = \frac{P_{Rotor}}{\frac{\rho}{2}\, v_1^3\, A_{Rotor}}$$

# vertical axis free stream turbine

**lift based design: <u>behaviour at low TSR:</u>**

**real power at low TSR:**




**-negative power at low velocities due to stall effects**
**- no lift but still drag**

**- turbine will not be able to start by itself**

**- but: drag based systems have good starting**

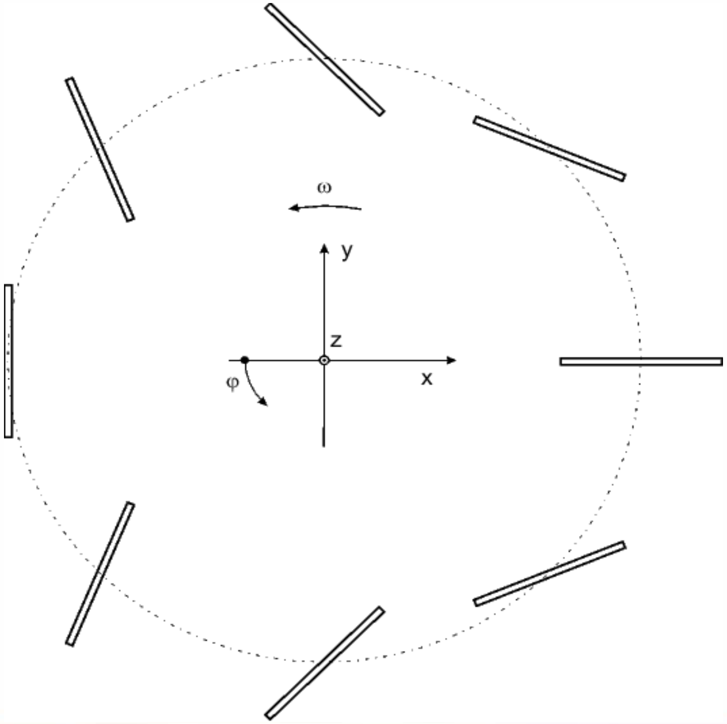
***Idea: use pitch control to use airfoils as drag areas!***

# vertical axis free stream turbine

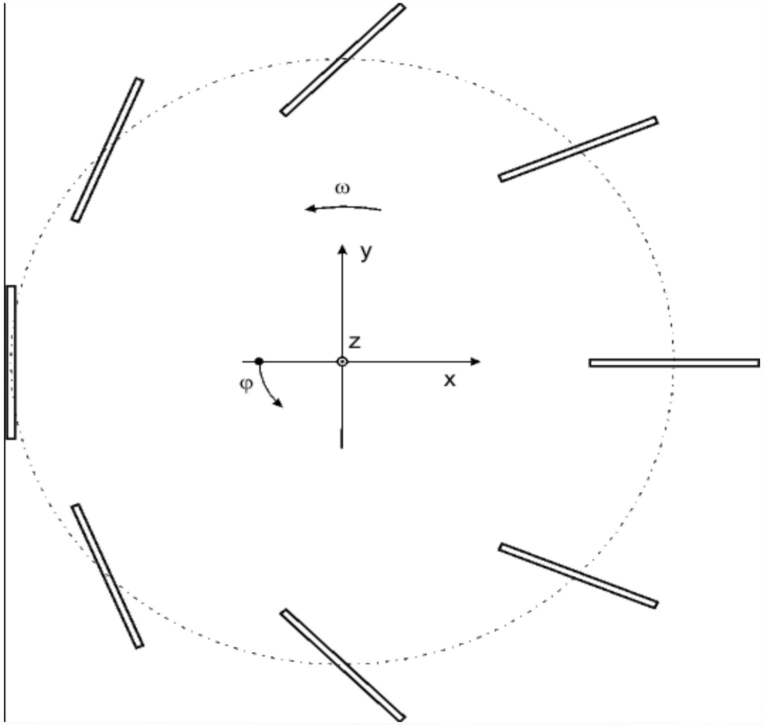
**pitch-variable design – use of drag:**

**- two possible kinematics:**

**anticlockwise**



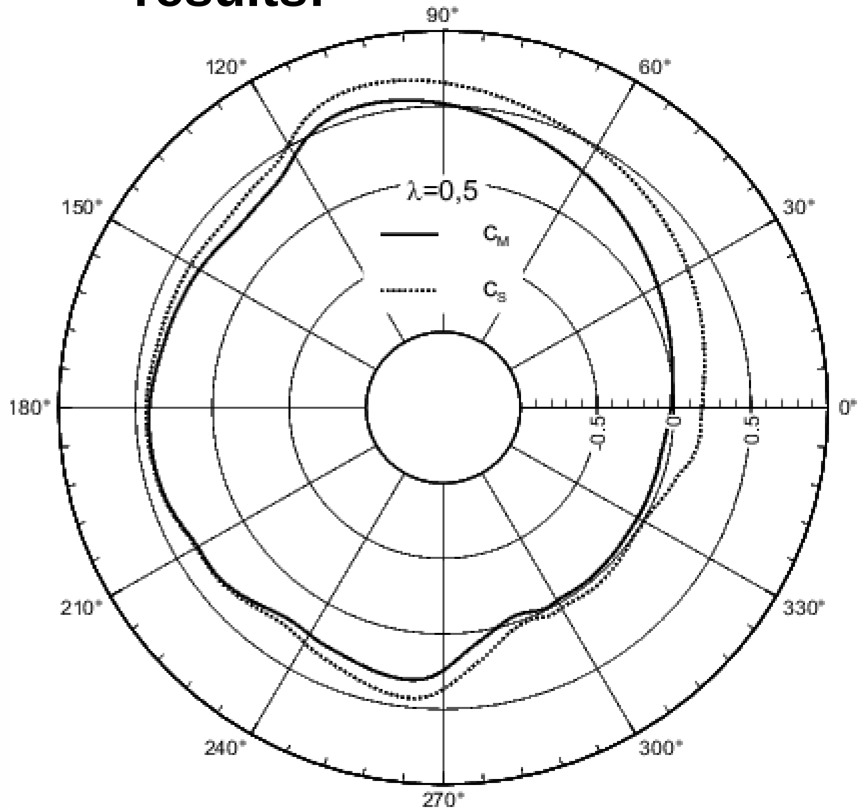

**clockwise**

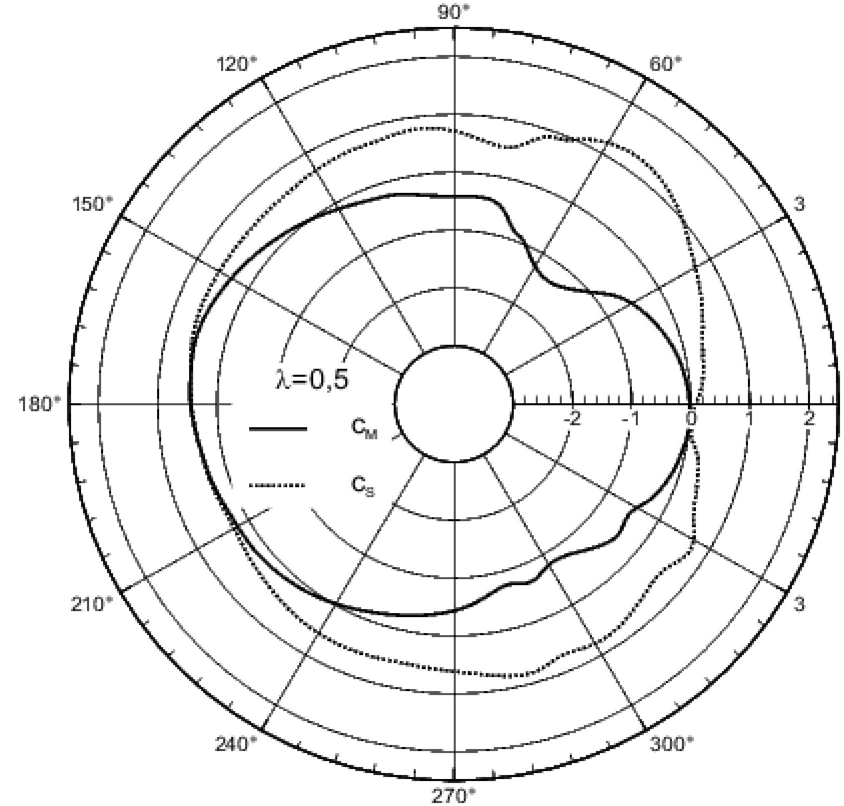


# vertical axis free stream turbine

## numeric simulation for drag based design at low TSR:

### results:



# vertical axis free stream turbine

**numeric simulation for drag based design at low TSR:**

**results: - videos**

# Investigations on current mills and wind turbines

**Thank you very much!**

# Appendix: Incompressibility

pressure diffence in front of the turbine and after it is negligible

$$\Delta p \approx 0$$

$$\Delta \rho \approx 0$$

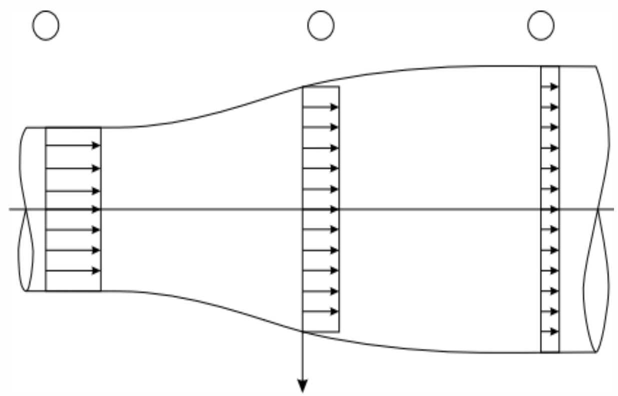
**incompressibility of the stream** through the turbine

$$\delta\rho / \delta t = \delta\rho / \delta x_i = 0$$

# Appendix: Froude-Rankine-Theorem

## Froude-Rankine-theorem :

conservation of momentum:



$$S = \dot{m}(v_1 - v_3)$$

Bernoulli 1 – 2

$$p_1 + \frac{\rho}{2}v_1^2 = p_{2-} + \frac{\rho}{2}v_2^2$$

Bernoulli 2 – 3

$$p_{2+} + \frac{\rho}{2}v_2^2 = p_3 + \frac{\rho}{2}v_3^2$$

pressure difference over the turbine:

$$p_{2-} - p_{2+} = \frac{\rho}{2}(v_1^2 - v_3^2)$$

$$S = A(p_{2-} - p_{2+}) = A\left(\frac{\rho}{2}(v_1^2 - v_3^2)\right) = \dot{m}(v_1 - v_3) = \rho v_2 A_2 (v_1 - v_3)$$

$$\boxed{v_2 = \frac{1}{2}\frac{(v_1 + v_3)(v_1 - v_3)}{(v_1 - v_3)} = \frac{v_1 + v_3}{2}}$$

# Appendix: Schmitz' theory

**Schmitz' theory of losses due to the angular momentum:**

$$c_{P_{Schmitz}} = \int_0^1 4\lambda \left(\frac{r}{R}\right)^2 \frac{\sin^3\left(\frac{2\arctan\left(\frac{R}{\lambda r}\right)}{3}\right)}{\sin^2\left(\arctan\left(\frac{R}{\lambda r}\right)\right)} \, d\left(\frac{r}{R}\right)$$