Line Integral

Integrated values of some parameter of an object along a straight line through the object.

$f(x,y)$	two-dimensional function representing a parameter of the object
$x\cos\theta + y\sin\theta = t$	line equation
$P_\theta(t) = \int_{(\theta,t)line} f(x,y)ds$	line integral

Projection

A projection is a set of line integrals.

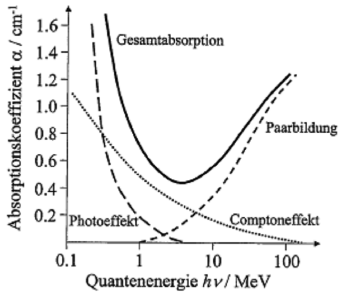
- 1st generation tomographs acquire a projection consisting of multiple parallel line integrals. This is repeated for different angles.
- 2nd generation tomographs require less translational steps as they take multiple line integrals measured along fans simultaneously.
- 3rd generation devices use only one fan beam projection for every angle.
- 4th generation tomographs rely also on fan beam projection whereas the have fixed detectors.

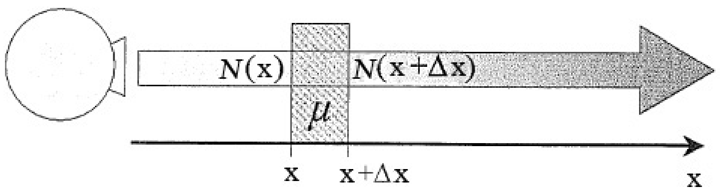
Measuring

The loss of beam intensity is measured. This loss happens due to:

- *Compton effect:* The x-ray photon interacts with a free, or loosely bound electron. It is deflected and losses energy.
- *Photoelectric absorption:* An x-ray photon losses all energy to a tightly bound inner electron in an atom.
- *Pair Production:* Can occur when x-ray photon energy is greater than 1.02 MeV. An electron and positron are created where the x-ray photon is annihilated. More important for PET.

All effects are energy-dependent.





The photon loss due to these effects is denoted by the attenuation coefficient μ which is given by:

$$(1)\ \frac{\Delta N}{N}\cdot\frac{1}{\Delta x} = -\mu .$$

The intensity of the x-ray beam after passing through Δx is given by:

$$(2)\ N(x)+\Delta x) = N(x) - \mu N(x)\Delta x$$

,where μ is assumed to be constant. As Δx goes to zero we obtain

$$(3)\ \lim_{\Delta x\to o}\frac{N(x+\Delta x)-N(x)}{\Delta x} = \frac{dN}{dx} = -\mu N(x).$$

Integrating on both sides

$$(4)\ \int\frac{dN}{N(x)} = -\mu\int dx$$

results in

$$(5)\ \ln|N| = -\mu x + C .$$

As N is always positive and under the initial condition that $N(0) = N_{in}$ we obtain

$$(6)\ N(x) = N_{in}\exp[-\mu x].$$

Replacing the constant μ by $\mu(x,y)$, (6) is enhanced to

$$(7)\ N = N_{in}\exp\left[-\int_{ray}\mu(x,y)ds\right]$$

,whereas all hitherto formulas are only true for monochromatic photons. Taking into account that x-ray consists of polychromatic photons (7) is altered to

$$(8)\ N = \int S_{in}(E)\exp\left[-\int_{ray}\mu(x,y,E)ds\right]dE .$$

Hounsfield scale

The values produced by a tomography are Hounsfield units (HU). Distilled water has been defined as 0 HU and air as -1000 HU. The relationship to the attenuation coefficient is given by:

$$H = \frac{\mu-\mu_{water}}{\mu_{water}}\times 1000$$