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Effiziente Algorithmen und Datenstrukturen I

Aufgabe 1 (10 Punkte)

n motorcyclists $M_1, M_2, \ldots, M_n$ start riding their bikes from a (straight) start line. At the start M_i and M_{i+1} are adjacent to each other. Each motorcyclist M_i starts at some angle ϕ_i and keeps riding in a straight line along this direction at a constant speed $s_i > 0$. Whenever a motorcyclist M_j comes across the path traversed by any other motorcyclist M_i, we say that M_i defeated M_j and in that case, M_j stops riding.

(a) We call the point where M_i defeats M_j as the point of ambush $A_{i,j} \in \mathbb{R}^2$. Show that if $A_{i',j'}$ is a point of ambush which occurs closest to the start line, then i' and j' are consecutive integers.

(b) Show how to enumerate in $O(n \log n)$ time, all events where one motorcyclist defeats another.

Aufgabe 2 (10 Punkte)

For any positive integer n, show a sequence of Fibonacci heap operations that creates a Fibonacci heap consisting of just one tree that is a linear chain of n nodes.

Aufgabe 3 (10 Punkte)

(a) Modify vEB trees to support duplicate keys.

(b) Modify vEB trees to support keys that have associated satellite data.

Aufgabe 4 (10 Punkte)

(a) Consider implementing Prim's algorithm using a vEB tree when the weight of each edge, $w_e \in \{1, 2, \ldots, \ell\}$. Use the fact that the keys are upper bounded to make sure that the largest key inserted into the priority queue is small (how small?). Analyze the running time of your algorithm. Compare this running time to an implementation using Fibonacci heaps. For which values of ℓ is the implementation using vEB trees more efficient?

(b) Do the same modification, analysis and comparison for Dijkstras algorithm.