

## 7.4 Augmenting Data Structures

Suppose you want to develop a data structure with:

- ▶ **Insert( $x$ ):** insert element  $x$ .
- ▶ **Search( $k$ ):** search for element with key  $k$ .
- ▶ **Delete( $x$ ):** delete element referenced by pointer  $x$ .
- ▶ **find-by-rank( $\ell$ ):** return the  $\ell$ -th element; return “error” if the data-structure contains less than  $\ell$  elements.

Augment an existing data-structure instead of developing a new one.

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1. choose an underlying data-structure
2. determine additional information to be stored in the underlying structure
3. verify/show how the additional information can be maintained for the basic modifying operations on the underlying structure.
4. develop the new operations

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**Goal: Design a data-structure that supports insert, delete, search, and find-by-rank in time  $\mathcal{O}(\log n)$ .**

1. We choose a red-black tree as the underlying data-structure.
2. We store in each node  $v$  the size of the sub-tree rooted at  $v$ .
3. We need to be able to update the size-field in each node without asymptotically affecting the running time of insert, delete, and search. We come back to this step later...

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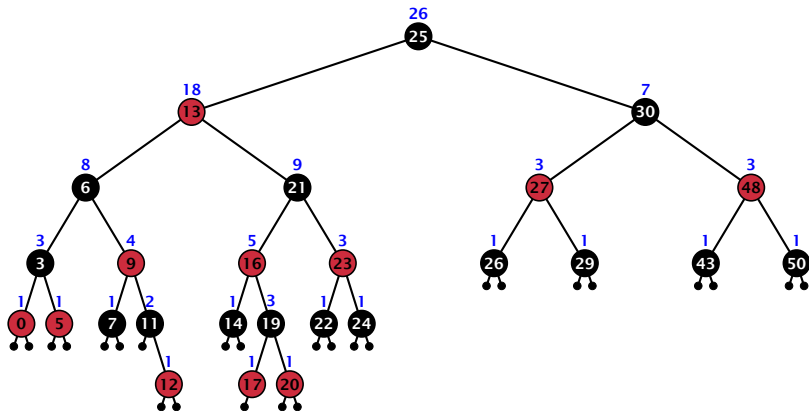
4. How does find-by-rank work?

Find-by-rank( $k$ ) := Select(root,  $k$ ) with

### Algorithm 15 Select( $x, i$ )

```
1: if  $x = \text{null}$  then return error
2: if left[ $x$ ]  $\neq$  null then  $r \leftarrow$  left[ $x$ ].size + 1 else  $r \leftarrow 1$ 
3: if  $i = r$  then return  $x$ 
4: if  $i < r$  then
5:     return Select(left[ $x$ ],  $i$ )
6: else
7:     return Select(right[ $x$ ],  $i - r$ )
```

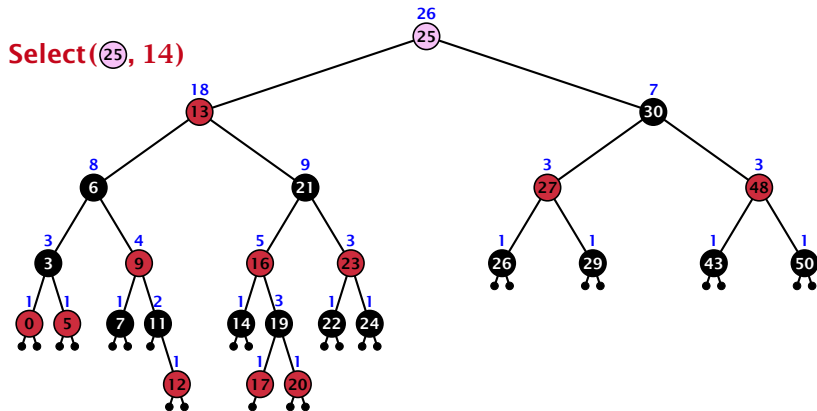
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### Find-by-rank:

- ▶ decide whether you have to proceed into the left or right sub-tree
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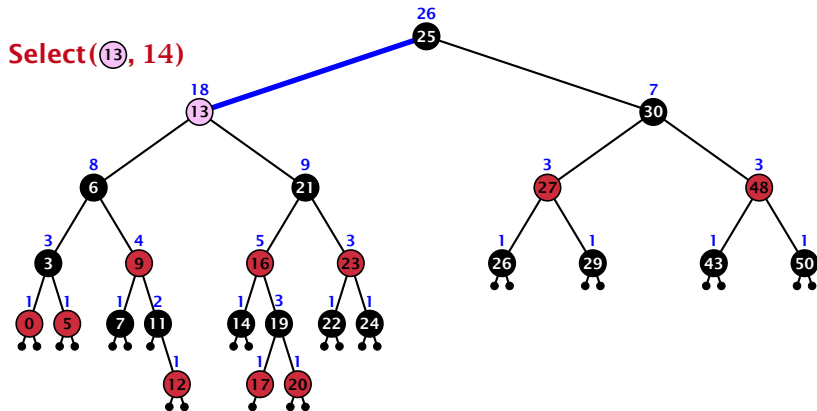
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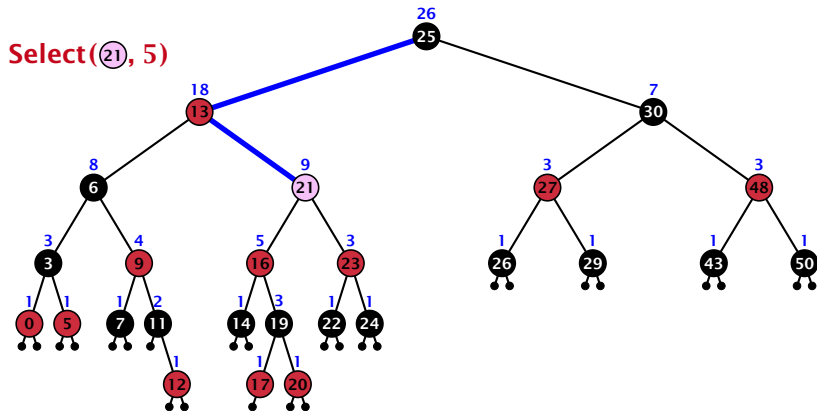
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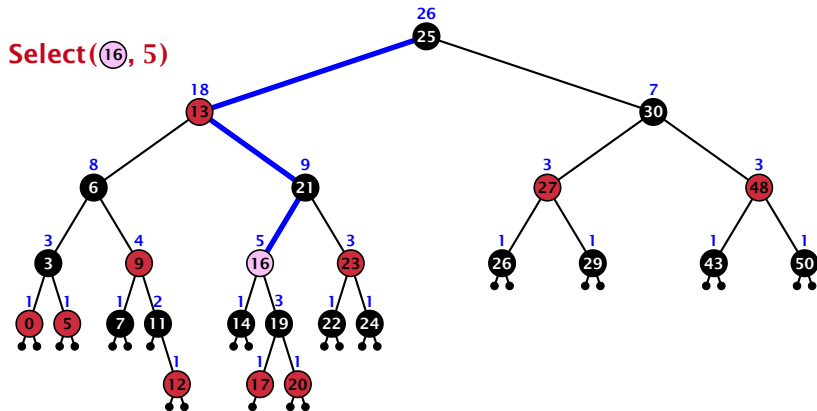
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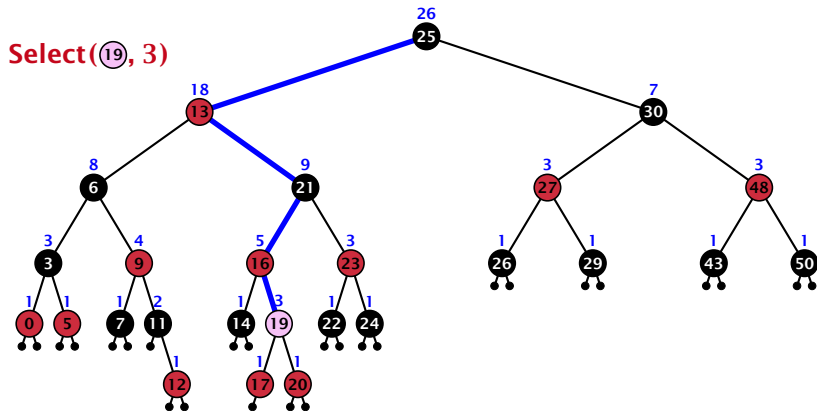
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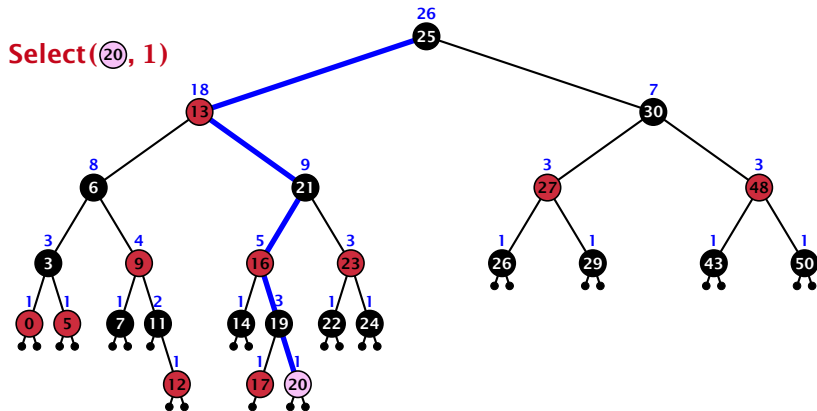


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3. How do we maintain information?

Search( $k$ ): Nothing to do.

Insert( $x$ ): When going down the search path increase the size field for each visited node. Maintain the size field during rotations.

Delete( $x$ ): Directly after splicing out a node traverse the path from the spliced out node upwards, and decrease the size counter on every node on this path. Maintain the size field during rotations.

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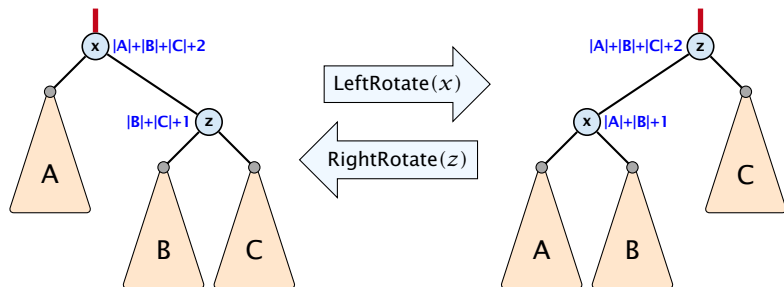
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## Rotations

The only operation during the fix-up procedure that alters the tree and requires an update of the size-field:



The nodes  $x$  and  $z$  are the only nodes changing their size-fields.

The new size-fields can be computed **locally** from the size-fields of the children.