
Effiziente Algorithmen und Datenstrukturen I

Aufgabe 1 (10 Punkte)

Prove the following statements:

1. $f(n) + g(n) \in \Omega(f(n))$
2. $f(n) \in O(g(n)) \Rightarrow f(n) + g(n) \in O(g(n))$
3. $f(n) \in o(g(n))$ and $g(n) \in O(h(n)) \Rightarrow h(n) \in \omega(f(n))$
4. $f(n) \in O(g(n))$ and $g(n) \in O(f(n)) \Leftrightarrow f(n) \in \Theta(g(n))$

State whether the following statement is true:

1. $\frac{1}{\Omega(n)} \subseteq O(\frac{1}{n})$

Aufgabe 2 (10 Punkte)

For constants $c, \epsilon > 0$ and $k > 1$, arrange the following functions of n in non-decreasing asymptotic order so that $f_i(n) = O(f_{i+1}(n))$ for two consecutive functions in your sequence. Also indicate whether $f_i(n) = \Theta(f_{i+1}(n))$ holds or not.

$$n^k, \sqrt{n}, 2^n, n^{1+\sin(n)}, \log(n!), n^{k+\epsilon}, n^n, n, n^k(\log n)^c, n!, n \log n, 3^n, n \log \log n, n \log(n^2)$$

Aufgabe 3 (10 Punkte)

Let $f(n)$ and $g(n)$ be asymptotically non-negative functions. Using the basic definition of Θ -notation, prove that $\max(f(n), g(n)) = \Theta(f(n) + g(n))$.

Aufgabe 4 (10 Punkte)

Show that for any real constants a and b , where $b > 0$,

$$(n + a)^b = \Theta(n^b)$$