

## Lemma 12 (Chernoff Bounds)

Let  $X_1, \dots, X_n$  be  $n$  *independent* 0-1 random variables, not necessarily identically distributed. Then for  $X = \sum_{i=1}^n X_i$  and  $\mu = E[X]$ ,  $L \leq \mu \leq U$ , and  $\delta > 0$

$$\Pr[X \geq (1 + \delta)U] < \left( \frac{e^\delta}{(1 + \delta)^{1+\delta}} \right)^U,$$

and

$$\Pr[X \leq (1 - \delta)L] < \left( \frac{e^{-\delta}}{(1 - \delta)^{1-\delta}} \right)^L,$$

### Lemma 13

For  $0 \leq \delta \leq 1$  we have that

$$\left( \frac{e^\delta}{(1+\delta)^{1+\delta}} \right)^U \leq e^{-U\delta^2/3}$$

and

$$\left( \frac{e^{-\delta}}{(1-\delta)^{1-\delta}} \right)^L \leq e^{-L\delta^2/2}$$

# Integer Multicommodity Flows

- ▶ Given  $s_i$ - $t_i$  pairs in a graph.
- ▶ Connect each pair by a path such that not too many paths use any given edge.

$$\begin{array}{ll} \min & W \\ \text{s.t.} & \forall i \quad \sum_{p \in \mathcal{P}_i} x_p = 1 \\ & \sum_{p: e \in p} x_p \leq W \\ & x_p \in \{0, 1\} \end{array}$$

# Integer Multicommodity Flows

## Randomized Rounding:

For each  $i$  choose one path from the set  $\mathcal{P}_i$  at random according to the probability distribution given by the Linear Programming Solution.

### Theorem 14

*If  $W^* \geq c \ln n$  for some constant  $c$ , then with probability at least  $n^{-c/3}$  the total number of paths using any edge is at most  $W^* + \sqrt{cW^* \ln n}$ .*

# Integer Multicommodity Flows

Let  $X_e^i$  be a random variable that indicates whether the path for  $s_i$ - $t_i$  uses edge  $e$ .

Then the number of paths using edge  $e$  is  $Y_e = \sum_i X_e^i$ .

$$E[Y_e] = \sum_i \sum_{p \in \mathcal{P}_i; e \in p} x_p^* = \sum_{p: e \in P} x_p^* \leq W^*$$

# Integer Multicommodity Flows

Choose  $\delta = \sqrt{(c \ln n)/W^*}$ .

Then

$$\Pr[Y_e \geq (1 + \delta)W^*] < e^{-W^* \delta^2/3} = \frac{1}{n^{c/3}}$$