

SS 2013

Komplexitätstheorie

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<http://www14.in.tum.de/lehre/2013SS/kt/>

Sommersemester 2013

Chapter 0 Organizational Matters

- Lectures:
 - 4SWS Tue 08:25–09:55 (MI HS2)
Fri 08:25–09:55 (MI 00.13.009A)
Compulsory elective in areas Algorithms and Scientific Computing, Informatics, Bioinformatics
Module no. IN2007
- Exercises/Tutorial:
 - 2SWS Central exercise: Thu 12:15–13:45 (03.11.018)
 - Tutor: Chris Pinkau
- Valuation:
 - 4V+2ZÜ, 8 ECTS Points
- Office hours:
 - Fri 11:00–12:00 and by appointment

- Tutorials:
 - Chris Pinkau, MI 03.09.057 (pinkau@in.tum.de)
Office hours: Tue 13:00–14:00
- Secretariat:
 - Mrs. Lissner, MI 03.09.052 (lissner@in.tum.de)

- Problem sets and final exam:
 - problem sets are made available on Tuesdays in class and/or on the course webpage
 - must be turned in a week later before class
 - are discussed in the tutorial
- Exam:
 - written exam, date: August 1, 2013, 11:30–14:00, room **Interims Hörsaal 101**
 - the final exam is closed book, no auxiliary means are permitted except for one sheet of DIN-A4 paper, handwritten by yourself
 - probably 12 problem sets




- Prerequisites:
 - Grundlagen: Algorithmen und Datenstrukturen (GAD)
 - Diskrete Wahrscheinlichkeitstheorie (DWT)
 - Effiziente Algorithmen und Datenstrukturen
 - Randomisierte Algorithmen
- Supplementary courses:
 - Approximationsalgorithmen
 - Internetalgorithmik
 - Quantenalgorithmen
 - ...
- Webpage:

<http://www.mayr.in.tum.de/lehre/2013SS/kt/>


1. Planned topics for the course

- 1 The computational model
- 2 \mathcal{NP} and \mathcal{NP} -completeness
- 3 Diagonalization
- 4 Space complexity
- 5 The polynomial hierarchy and alternation
- 6 Boolean circuits
- 7 (Randomized computation)
- 8 Interactive proofs
- 9 Cryptography
- 10 ...

2. Literature

-  Sanjeev Arora, Boaz Barak:
Computational Complexity — A Modern Approach,
Cambridge University Press: Cambridge-New York-Melbourne, 2009
-  Giorgio Ausiello, Pierluigi Crescenzi, Giorgio Gambosi, Viggo Kann, Alberto Marchetti-Spaccamela, Marco Protasi:
Complexity and approximation — Combinatorial optimization problems and their approximability properties,
Springer-Verlag: Berlin-Heidelberg, 1999
-  José L. Balcázar, Josep Díaz, Joaquim Gabarró:
Structural Complexity I (and II),
EATCS Monographs on Theoretical Computer Science, Springer-Verlag:
Berlin-Heidelberg, 1995

-  **Christos H. Papadimitriou:**
Computational Complexity,
Addison-Wesley Publishing Company: London-Amsterdam-New York, 1994
-  **Christos H. Papadimitriou, Kenneth Steiglitz:**
Combinatorial optimization: Algorithms and complexity,
Prentice-Hall, Englewood Cliffs, NJ, 1982
-  **Karl Rüdiger Reischuk:**
Komplexitätstheorie — Band I: Grundlagen,
B.G. Teubner: Stuttgart-Leipzig, 1999
-  **Michael Sipser:**
Introduction to the Theory of Computation,
International Edition, Thomson Course Technology:
Australia-Canada-Mexico-Singapore-Spain-United Kingdom-United States, 2006

 Ingo Wegener:
The complexity of Boolean functions,
Wiley-Teubner Series in Computer Science: Stuttgart-Chichester-New York, 1987,
http://eccc.hpi-web.de/static/books/The_Complexity_of_Boolean_Functions/

Further relevant research papers will be made available during the course.

3. Notational conventions

We use standard notation and basic concepts, as detailed e.g., in the introductory course on

Discrete Structures, IN0015

<http://wwwmayr.in.tum.de/lehre/2012WS/ds/index.html.en>

Chapter I The Computational Model

1. Some basic concepts

See



Sanjeev Arora, Boaz Barak:

Computational Complexity — A Modern Approach,

p. 9–12, Cambridge University Press: Cambridge-New York-Melbourne, 2009

2. Turing Machines

2.1 The Model

2.2 Robustness

2.3 Gödel Numbers and Universal TMs

See

 [Sanjeev Arora, Boaz Barak:](#)
Computational Complexity — A Modern Approach,
p. 13–23, Cambridge University Press: Cambridge–New York–Melbourne, 2009

2.4 Universal Simulation

 [Sanjeev Arora, Boaz Barak:](#)
Computational Complexity — A Modern Approach,
p. 33–36, Cambridge University Press: Cambridge-New York-Melbourne, 2009

2.5 Noncomputable Functions, the Halting Problem

2.6 Deterministic Time and the Class \mathcal{P}

See

 [Sanjeev Arora, Boaz Barak:](#)
Computational Complexity — A Modern Approach,
p. 23–29, Cambridge University Press: Cambridge-New York-Melbourne, 2009

Chapter II \mathcal{NP} and \mathcal{NP} -completeness

1. The class \mathcal{NP}

 Sanjeev Arora, Boaz Barak:
Computational Complexity — A Modern Approach,
p. 37–39, Cambridge University Press: Cambridge–New York–Melbourne, 2009

1.1 Relation between \mathcal{P} and \mathcal{NP}

1.2 Non-deterministic Turing Machines

2. Reducibility and \mathcal{NP} -completeness

 Sanjeev Arora, Boaz Barak:
Computational Complexity — A Modern Approach,
p. 39–42, Cambridge University Press: Cambridge–New York–Melbourne, 2009

3. Cook-Levin theorem

3.1 Boolean formulae and CNF

3.2 The Cook-Levin theorem

 [Sanjeev Arora, Boaz Barak:](#)
Computational Complexity — A Modern Approach,
p. 42–47, Cambridge University Press: Cambridge–New York–Melbourne, 2009

4. The web of reductions

-  Sanjeev Arora, Boaz Barak:
Computational Complexity — A Modern Approach,
p. 47–53, Cambridge University Press: Cambridge-New York-Melbourne, 2009

5. Decision versus search

6. $\text{co}\mathcal{NP}$, EXP, and NEXP

7. Some implications

See

 [Sanjeev Arora, Boaz Barak:](#)
Computational Complexity — A Modern Approach,
p. 53–59, Cambridge University Press: Cambridge-New York-Melbourne, 2009

Also see



Michael R. Garey, David S. Johnson:

Computers and Intractability: A Guide to the Theory of \mathcal{NP} -completeness,
W.H. Freeman and Company: New York-San Francisco, 1979

and the websites (among many others)

List of Complexity Classes (Wikipedia)
Complexity Zoo
 \mathcal{NP} -complete Problems (Wikipedia)
 \mathcal{NP} -completeness Columns
A Compendium of NP Optimization Problems

7.1 \mathcal{NP} -complete problems cannot be sparse

See



Holenstein, Thomas

Complexity Theory,


p. 4–4, Script, ETH Zürich, 2010

Chapter III Diagonalization

1. Time and space hierarchy

See

 [Sanjeev Arora, Boaz Barak:](#)
Computational Complexity — A Modern Approach,
p. 63–64, Cambridge University Press: Cambridge-New York-Melbourne, 2009

 [Holenstein, Thomas](#)
Complexity Theory,
p. 2–3, Script, ETH Zürich, 2010

2. Non-deterministic time hierarchy

3. Ladner's theorem

See

 [Sanjeev Arora, Boaz Barak:](#)
Computational Complexity — A Modern Approach,
p. 65–67, Cambridge University Press: Cambridge–New York–Melbourne, 2009

 [Lance Fortnow, Bill Gasarch:](#)
[Computational Complexity Blog](#)
[Two Proofs of Ladner's Theorem](#), 2003

4. Oracle machines and limits of diagonalization

See

 [Sanjeev Arora, Boaz Barak:](#)
Computational Complexity — A Modern Approach,
p. 68–71, Cambridge University Press: Cambridge–New York–Melbourne, 2009

On a less formal side, also see

[Relativization and Multiple Choice Exams](#)

Chapter IV Space Complexity

1. Configuration graphs

2. Some space complexity classes

See

 [Sanjeev Arora, Boaz Barak:](#)
Computational Complexity — A Modern Approach,
p. 75–78, Cambridge University Press: Cambridge–New York–Melbourne, 2009

3. PSPACE completeness

3.1 Savitch's theorem

See



Sanjeev Arora, Boaz Barak:

Computational Complexity — A Modern Approach,

p. 78–82, Cambridge University Press: Cambridge-New York-Melbourne, 2009

3.2 PSPACE and strategies for game playing

4. \mathcal{NL} -completeness


4.1 Certificate definition of \mathcal{NL} : Read-once certificates


4.2 $\mathcal{NL} = \text{co}\mathcal{NL}$

See


 [Sanjeev Arora, Boaz Barak:](#)
Computational Complexity — A Modern Approach,
p. 83–88, Cambridge University Press: Cambridge–New York–Melbourne, 2009

Further references:

 Larry J. Stockmeyer, Albert R. Meyer:
Word problems requiring exponential time,
Proceedings of the 5th Symposium on Theory of Computing, p. 1–9 (1973)
This paper contains some important PSPACE-completeness results.

 Albert R. Meyer, Larry J. Stockmeyer:
The equivalence problem for regular expressions with squaring requires exponential space,
Proceedings of the 13th Annual Symposium on Switching and Automata Theory,
p. 125–129 (1972)
This paper contains an EXPSPACE-completeness result.

And here an \mathcal{NL} -machine based proof for $\mathcal{NL} = \text{co}\mathcal{NL}$:

 Holenstein, Thomas
Complexity Theory,
p. 13–14, Script, ETH Zürich, 2010

Chapter V The Polynomial Hierarchy and Alternation

1. The class Σ_2^P
2. The polynomial hierarchy
3. Alternating Turing machines

See

 [Sanjeev Arora, Boaz Barak:](#)
Computational Complexity — A Modern Approach,
p. 91–96, Cambridge University Press: Cambridge-New York-Melbourne, 2009

4. Time versus alternations: Time-space tradeoffs for SAT

See

 [Sanjeev Arora, Boaz Barak:](#)
Computational Complexity — A Modern Approach,
p. 96–98, Cambridge University Press: Cambridge-New York-Melbourne, 2009

5. Defining the hierarchy via oracle machines

See

 [Sanjeev Arora, Boaz Barak:](#)
Computational Complexity — A Modern Approach,
p. 98–99, Cambridge University Press: Cambridge-New York-Melbourne, 2009

Chapter VI Boolean Circuits

1. Boolean circuits and $\mathcal{P}_{/poly}$
2. Uniformly generated circuits
3. Turing machines that take advice

See



Sanjeev Arora, Boaz Barak:

Computational Complexity — A Modern Approach,
p. 101–106, Cambridge University Press: Cambridge-New York-Melbourne, 2009



Leighton, F. Thomson:


Introduction to Parallel Algorithms and Architectures: Arrays, Trees, Hypercubes,
Morgan Kaufmann: San Mateo, 1992

4. $\mathcal{P}_{/\text{poly}}$ and \mathcal{NP} : Karp-Lipton Theorem

5. Circuit lower (and upper) bounds

See

 Sanjeev Arora, Boaz Barak:
Computational Complexity — A Modern Approach,
p. 106–107, Cambridge University Press: Cambridge-New York-Melbourne, 2009

 G.S. Frandsen and P. Bro Miltersen:
Reviewing bounds on the circuit size of the hardest functions,
IPL **95** (2005), p. 354–357

6. Non-uniform hierarchy theorem

7. Finer gradations among circuit classes

8. Circuits of exponential size

See

 [Sanjeev Arora, Boaz Barak:](#)
Computational Complexity — A Modern Approach,
p. 108–113, Cambridge University Press: Cambridge-New York-Melbourne, 2009

Chapter VII Randomized Computation

1. Probabilistic Turing Machines
2. Some examples of PTMs
3. One-sided and zero-sided error: \mathcal{RP} , $\text{co}\mathcal{RP}$, ZPP

See

 [Sanjeev Arora, Boaz Barak:](#)
Computational Complexity — A Modern Approach,
p. 115–120, Cambridge University Press: Cambridge-New York-Melbourne, 2009

4. The robustness of our definitions

See

 [Sanjeev Arora, Boaz Barak:](#)
Computational Complexity — A Modern Approach,
p. 121–126, Cambridge University Press: Cambridge-New York-Melbourne, 2009

5. $\text{BPP} \subseteq \mathcal{P}_{/\text{poly}}$

6. BPP is in \mathcal{PH}

7. Randomized reductions

8. Randomized space-bounded computation

See

 [Sanjeev Arora, Boaz Barak:](#)
Computational Complexity — A Modern Approach,
p. 126–146, Cambridge University Press: Cambridge-New York-Melbourne, 2009

Chapter VIII Interactive Proofs

1. Interactive proofs: Some variations

1.1 Interactive proofs with deterministic verifier and prover

2. The class IP: Probabilistic verifier

3. Interactive proof for graph nonisomorphism

See

 [Sanjeev Arora, Boaz Barak:](#)
Computational Complexity — A Modern Approach,
p. 147–151, Cambridge University Press: Cambridge-New York-Melbourne, 2009

4. Public coins and AM

4.1 Simulating private coins

4.2 Set lower bound protocol

4.3 Some properties of \mathcal{IP} and AM

See

 [Sanjeev Arora, Boaz Barak:](#)
Computational Complexity — A Modern Approach,
p. 151–156, Cambridge University Press: Cambridge-New York-Melbourne, 2009

4.4 Can GI be \mathcal{NP} -complete?

5. $IP = PSPACE$

5.1 Arithmetization

5.2 Interactive protocol for $\#SAT_D$

See

 [Sanjeev Arora, Boaz Barak:](#)
Computational Complexity — A Modern Approach,
p. 157–159, Cambridge University Press: Cambridge-New York-Melbourne, 2009

5.3 Protocol for TQBF

See

-  [Sanjeev Arora, Boaz Barak:](#)
Computational Complexity — A Modern Approach,
p. 159–161, Cambridge University Press: Cambridge-New York-Melbourne, 2009
-  [Holenstein, Thomas](#)
Complexity Theory,
p. 64–69, Script, ETH Zürich, 2010

6. The power of the prover

7. Multiprover interactive proofs

8. Program checking

See

 [Sanjeev Arora, Boaz Barak:](#)
Computational Complexity — A Modern Approach,
p. 161–170, Cambridge University Press: Cambridge-New York-Melbourne, 2009

9. Outlook, More Topics in Complexity Theory

Among numerous other topics, see e.g. this on the GCT program:



Mulmuley, Ketan D.:

The GCT Program Toward the \mathcal{P} vs. \mathcal{NP} Problem,

Comm. ACM **55**, 6, p. 100–109, ACM Press: New York, 2012

For a blurb on one of the authors of the textbook for this course, see



Hyman, Paul:

An Influential Theoretician,

Comm. ACM **55**, 6, p. 24, ACM Press: New York, 2012