
Efficient Algorithms and Datastructures I

Question 1 (10 Points)

An airline has n flight legs that it wishes to service by the fewest possible planes. To do so, it must determine the most efficient way to combine these legs into flight schedules. The starting time for flight i is a_i and the finishing time is b_i . A plane requires r_{ij} hours to travel from the point of destination of flight i to the point of origin of flight j . Show how to solve this problem efficiently by formulating it as a min-cost flow problem.

Question 2 (10 Points)

The Mathematics and Computer Science department has n faculty members f_1, f_2, \dots, f_n who will offer n courses c_1, c_2, \dots, c_n in the coming semester and each faculty member will teach exactly one course. Each faculty member chooses two courses he (or she) would like to teach, and ranks them according to his (or her) preference (rank 1 indicates higher preference and rank 2 indicates lower preference).

- We say that a course assignment is a *feasible* assignment if every faculty member teaches a course within his (or her) preference list. How would you efficiently determine whether the department can find a feasible assignment?
- We say that a feasible assignment is an *optimal assignment* if it maximizes the number of faculty members assigned to their most preferred course. Suggest an efficient algorithm for determining an optimal assignment and analyze its complexity.

Question 3 (10 Points)

A shipping company wants to phase out a fleet of s (homogeneous) cargo ships over a period of p years. Its objective is to maximize its cash assets at the end of the p years by considering the possibility of prematurely selling ships and temporarily replacing them by charter ships. The company faces a known nonincreasing demand for ships. Let $d(i)$ denote the demand of ships in year i . Each ship earns a revenue of r_k units in period k . At the beginning of year k , the company can sell any ship that it owns, accruing a cash inflow of s_k dollars. If the company does not own sufficiently many ships to meet its demand, it must hire additional charter ships. Let h_k denote the cost of hiring a ship for the k th year. The shipping company wants to meet its commitments and at the same time maximize the cash assets at the end of the p th year. Formulate this problem as a minimum cost flow problem.

Question 4 (10 Points)

The dean needs to assign a group of n faculty members to be chairs on n committees. Each faculty member proposes, in decreasing order of preference, a list of three committees that he or she would like to chair. We want to determine whether there exists a *satisfiable assignment* (one that assigns the faculty to the committees so that each faculty member obtains a job on his or her list). If some satisfiable assignment is possible, we want to find the assignment that maximizes the number of faculty with their most preferred committee chair, and further, among such assignments, the assignment that maximizes the number of faculty with their second most preferred committee chair. Show how to solve this problem by solving a single assignment problem.