7.4 Augmenting Data Structures

Suppose you want to develop a data structure with:

- **Insert(x)**: insert element x.
- **Search(k)**: search for element with key k.
- **Delete(x)**: delete element referenced by pointer x.
- **find-by-rank(ℓ)**: return the ℓ-th element; return “error” if the data-structure contains less than ℓ elements.

Augment an existing data-structure instead of developing a new one.

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2. determine additional information to be stored in the underlying structure
3. verify/show how the additional information can be maintained for the basic modifying operations on the underlying structure.
4. develop the new operations

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Goal: Design a data-structure that supports insert, delete, search, and find-by-rank in time $\mathcal{O}(\log n)$.

1. We choose a red-black tree as the underlying data-structure.
2. We store in each node v the size of the sub-tree rooted at v.
3. We need to be able to update the size-field in each node without asymptotically affecting the running time of insert, delete, and search. We come back to this step later...

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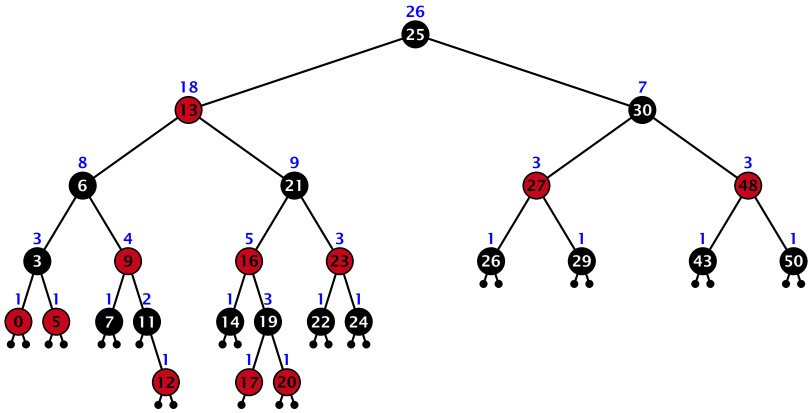
4. How does find-by-rank work?
   Find-by-rank(k) := Select(root,k) with

Algorithm 15 Select(x, i)

```
1: if $x = \text{null}$ then return error
2: if $\text{left}[x] \neq \text{null}$ then $r \leftarrow \text{left}[x].\text{size} + 1$ else $r \leftarrow 1$
3: if $i = r$ then return $x$
4: if $i < r$ then
5:         return Select$(\text{left}[x], i)$
6: else
7:         return Select$(\text{right}[x], i - r)$
```

Select(x, i)



Find-by-rank:

- decide whether you have to proceed into the left or right sub-tree
- adjust the rank that you are searching for if you go right

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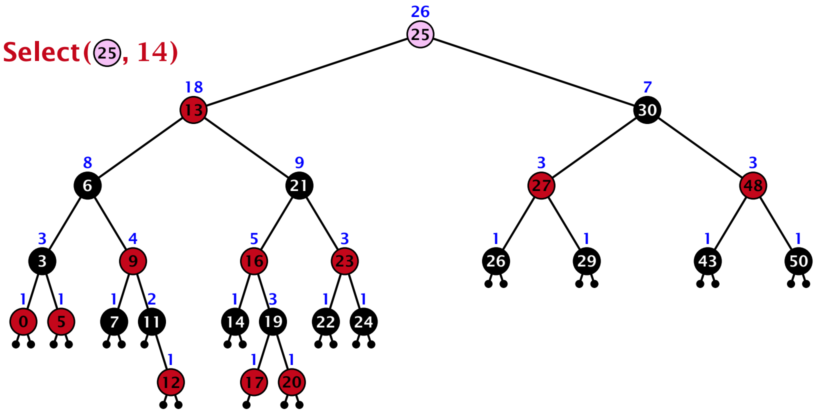

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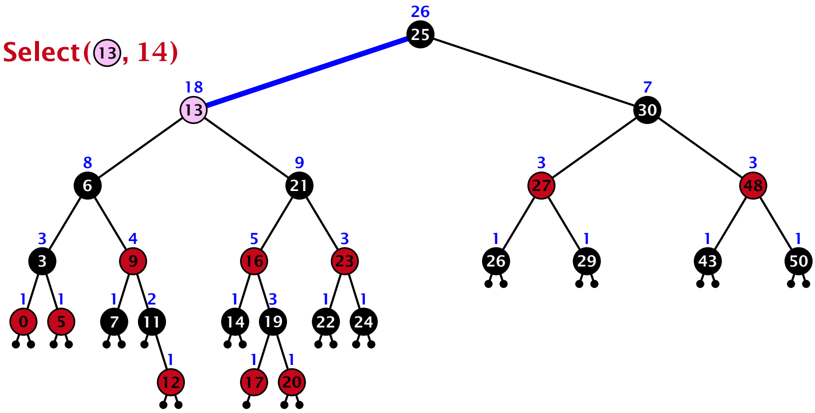

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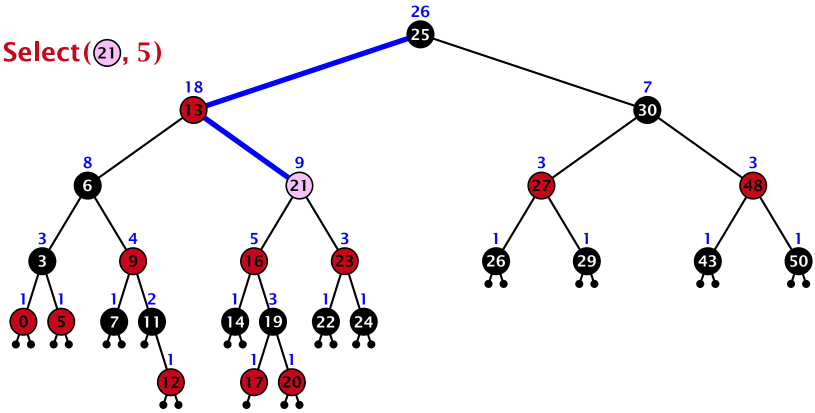

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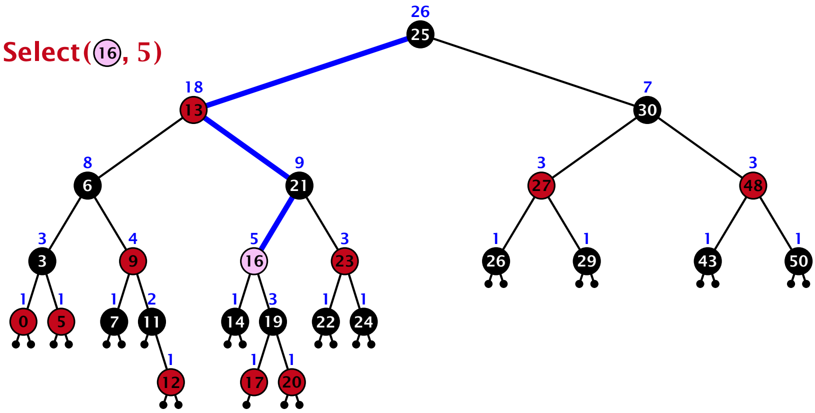

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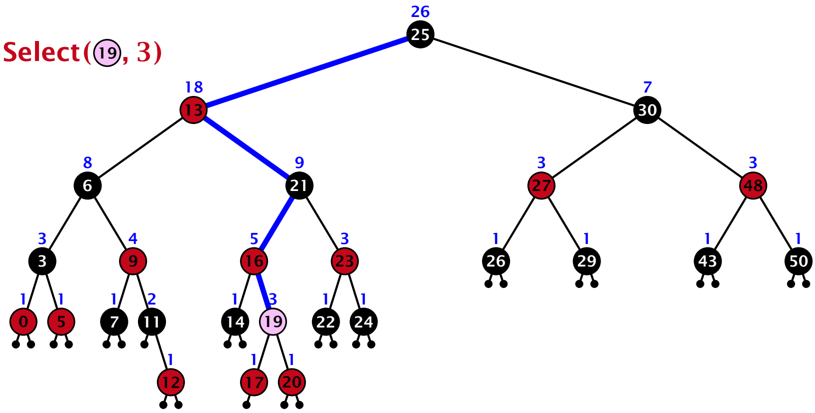

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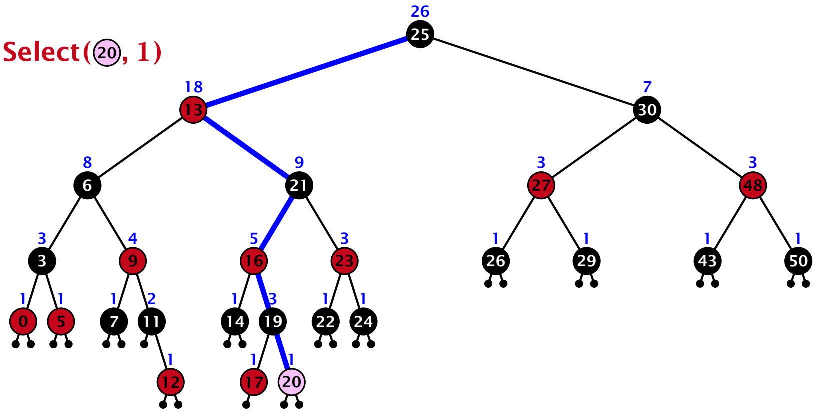

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Goal: Design a data-structure that supports insert, delete, search, and find-by-rank in time $\mathcal{O}(\log n)$.

3. How do we maintain information?

$\textbf{Search}(k)$: Nothing to do.

$\textbf{Insert}(x)$: When going down the search path increase the size field for each visited node. Maintain the size field during rotations.

$\textbf{Delete}(x)$: Directly after splicing out a node traverse the path from the spliced out node upwards, and decrease the size counter on every node on this path. Maintain the size field during rotations.

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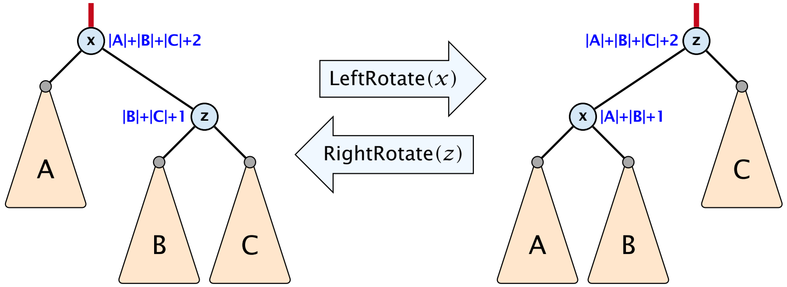
Rotations

The only operation during the fix-up procedure that alters the tree and requires an update of the size-field:



The nodes x and z are the only nodes changing their size-fields.

The new size-fields can be computed locally from the size-fields of the children.