Technische Universität München
Fakultät für Informatik
Lehrstuhl für Effiziente Algorithmen
Prof. Dr. Harald Räcke
Chintan Shah

Wintersemester 2014-15
Assignment 10
December 15, 2014

Efficient Algorithms and Datastructures I

Question 1 (10 Points)

Show that any sequence of m MAKE-SET, FIND-SET, and LINK operations, where all the LINK operations appear before any of the FIND-SET operations, takes only $O(m)$ time if both path compression and union by rank are used. What happens in the same situation if only the path-compression heuristic is used?

Question 2 (10 Points)

In cuckoo hashing, we double or halve the hash table size and rehash all the elements to ensure that the fill-factor is within a range. Explain formally how we can amortize this cost for rehashing against cost for insertions and deletions, by setting up a potential function.

Question 3 (10 Points)

Suppose instead of using decimal or dual representation of numbers, we represent them in binary over the basis of Fibonacci numbers. That is, the bit-string $(X_k, X_{k-1}, \cdots, X_1)_F$ represents the number $n = \sum_{i=1}^{k} X_i \cdot F_i$, where F_i denotes the ith Fibonacci number ($F_1 = F_2 = 1$ and $F_i = F_{i-1} + F_{i-2}$ for $i \geq 3$). For example, $(31)_{10}$ can be represented by the bit string $(10100100)_F$ since $F_8 + F_6 + F_3 = (21)_{10} + (8)_{10} + (2)_{10} = (31)_{10}$, and also by the bit string $(10011011)_F$ since $F_8 + F_5 + F_4 + F_2 + F_1 = (21)_{10} + (5)_{10} + (3)_{10} + (1)_{10} + (1)_{10} = (31)_{10}$.

(a) Argue that we can represent any number $n \in \mathbb{N}_0$ like this.

(b) Describe an algorithm which performs increment and decrement operations in this representation in constant amortized time (starting from 0). Assume that flipping each bit requires one unit of work.
(*Hint*: Use a potential function that assigns potential depending on whether consecutive pairs of bits are similar. For example, if bit i and bit $i + 1$ are equal/different, you may say they contribute one unit to the potential. Make sure that the potential of $(0)_F$ is zero units.)