Technische Universität München
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Summer Term 2015
Problem Set 11
June 29, 2015

Complexity Theory

Due date: July 6, 2015 **before** *class!*

Problem 1 (10 Points)

A language $L \subseteq \{0,1\}^*$ is *sparse* if there is a polynomial p such that $|L \cap \{0,1\}^n| \leq p(n)$ for every $n \in \mathbb{N}$.
Show that every sparse language is in $\mathcal{P}_{/poly}$.

Problem 2 (10 Points)

The language Connected from Problem Set 1 is in $\mathcal{P}$, hence it can be computed with a logspace-uniform circuit family. Describe the construction of such a circuit, when the input is given by the adjacency matrix A of a graph G, i.e. the input variables are the n^2 entries of A.

Problem 3 (10 Points)

Recall that the definition of **BPP** has the success probability $\frac{2}{3}$. Consider a definition with success probability $\frac{1}{2}$. The big issue would be the case where there are as many accepting branches as there are rejecting branches. Argue how to alter the PTM to ensue that this case can never occur.

Problem 4 (10 Points)

Error reduction for **RP**: Let $L \subseteq \{0,1\}^*$ be such that there exists a polynomial-time PTM M satisfying for every $x \in \{0,1\}^*$: $x \in L \implies Pr[M(x) = 1] \geq n^{-c}$ and $x \notin L \implies Pr[M(x) = 1] = 0$.
Prove that that for every $d > 0$ there exists a polynomial-time PTM M' such that for every $x \in \{0,1\}^*$: $x \in L \implies Pr[M'(x) = 1] \geq 1 - 2^{-n^d}$ and $x \notin L \implies Pr[M'(x) = 1] = 0$.