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Problem set 6
May 25, 2015
Summer Semester 2015

Online and Approximation Algorithms

Due June 1, 2015 before class!

Exercise 1 (Navigation on a line - 10 points)

Propose a randomized algorithm for the navigation problem on a line and show that it is 7-competitive.

Exercise 2 (Navigation on multiple paths - 10 points)

We are standing at a point s which is the beginning of k paths and we want to reach a target t. We know that t is at distance d from s but we do not know on which path. Our objective is to reach t by travelling the minimum distance possible. Show that the competitive ratio of any deterministic algorithm is at least $2k-1$ and that the competitive ratio of any randomized algorithm is at least k.

Exercise 3 (Exploration on cellular environments - 10 points)

Consider a robot which moves in a rectangular grid environment similar to a chessboard. Each cell of the environment is either *free* and can be visited by the robot, or *blocked* and unaccessible by the robot. In one step, the robot is in some cell and it moves in one of the 8 neighboring cells. Of course, the new cell must be free. Starting from a cell s, we want to visit all the empty cells of the environment and return back to s assuming that the environment is not known by the robot in advance. The objective is to perform a minimum number of steps. Propose an online algorithm and show that it is 2-competitive.

Exercise 4 (Spiral - 10 points)

Image yourself standing on a 2-dimensional grid searching for a point $t = (t_v, t_h)$ where t_v and t_h are the vertical and horizontal distances from your current position. You do know that t is finite, however you do not know the exact values for t_v and t_h. Develop an algorithm for reaching t and prove its competitiveness.