Technische Universität München
Fakultät für Informatik
Lehrstuhl für Effiziente Algorithmen
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Wintersemester 2014-15
Final Exam
6th February, 2016

Effiziente Algorithmen und Datenstrukturen I

Last Name	First Name	Matrikel No.
....	....	....

Hall	Seat No.	Signature
....	....	....

General Information for the Examination

- Please keep your identity card readily available.
- Do not use pencils. Do not write in red or green ink.
- You are not allowed to use anything except a single sided handwritten A4 paper.
- Verify that you have received 16 printed sides (check page numbers).
- Attempt all questions. You have 150 minutes to answer the questions.

Left Examination Hall from ...... to ...... / from ...... to ......
Submitted Early at ......
Special Notes:

	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	Σ	Examiner
Max.	3	3	2	3	4	3	7	4	5	6	40	
1st												
2nd												

Question 1 (3 Points)

(a) Give the generating function of the sequence $a_n = n + 1$.

(b) Show that $2^n \in o(3^n)$.

(c) Solve the following recurrence: $T(n) = 2T(\frac{n}{4}) + \sqrt{n}$

Question 2 (3 Points)

For constants $c, \epsilon > 0$ and $k > 1$, arrange the following functions of n in non-decreasing asymptotic order so that $f_i(n) = O(f_{i+1}(n))$ for two consecutive functions in your sequence. Also indicate whether $f_i(n) = \Theta(f_{i+1}(n))$ holds or not.

$$\log(n!), n^{k+\epsilon}, n, n^k(\log n)^c, n \log\log n, n \log(n^2)$$

Question 3 (2 Points)

(a) Suppose you have an addressable minheap which supports the following operations:

(i) handle INSERT(element x)

(ii) element DELETE-MIN()

(iii) void CHANGE-PRIORITY(handle h, new-priority)

How could you combine these operations to define a DELETE(handle h) operation?

(b) Suppose you have an addressable minheap which supports the following operations:

(i) handle INSERT(element x)

(ii) element DELETE-MIN()

(iii) void DELETE(handle h)

How could you combine these operations to define a CHANGE-PRIORITY(handle h, new-priority) operation?

Question 4 (3 Points)

A sequence of n operations is performed on a data structure which supports a single operation. The i-th call of this operation costs i if i is an exact power of 2, and 1 otherwise. Determine the amortized cost per operation.

Question 5 (4 Points)

The H-graph of order 0 is just a single node. The H-graphs of order $1, 2$, 3, and 4 are depicted in Figure 1, Figure 2, Figure 3, and Figure 4, respectively. Let $f(\ell)$ denote the number of vertices of an H-graph of order ℓ. Develop a recurrence relation for f and solve your relation using techniques from the lecture.

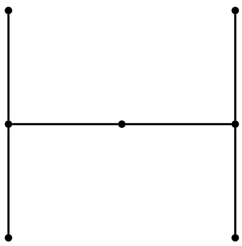


Abbildung 1: An H-graph of order 1

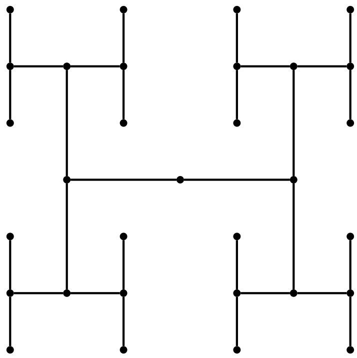


Abbildung 2: An H-graph of order 2

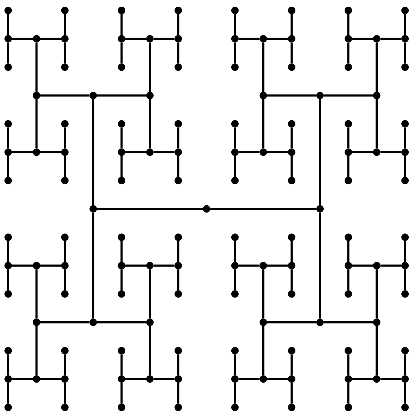


Abbildung 3: An H-graph of order 3

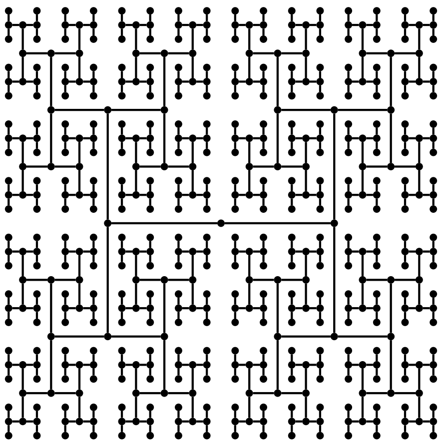
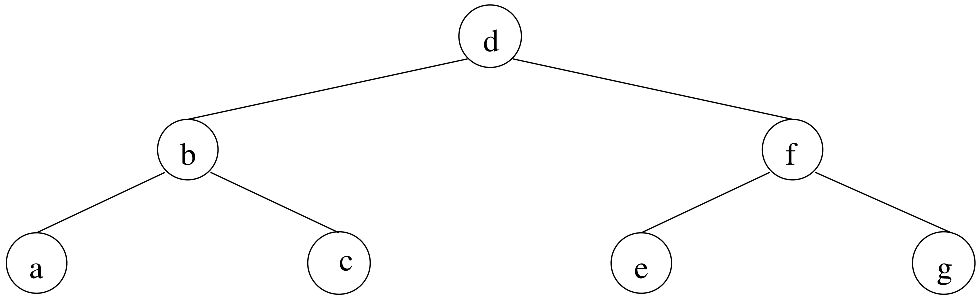


Abbildung 4: An H-graph of order 4

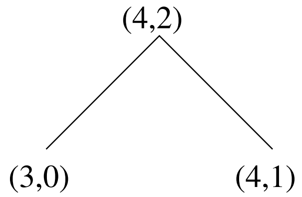
Question 6 (3 Points)

Access the characters g, c, e sequentially in the following splay tree and update the splay tree after each access.

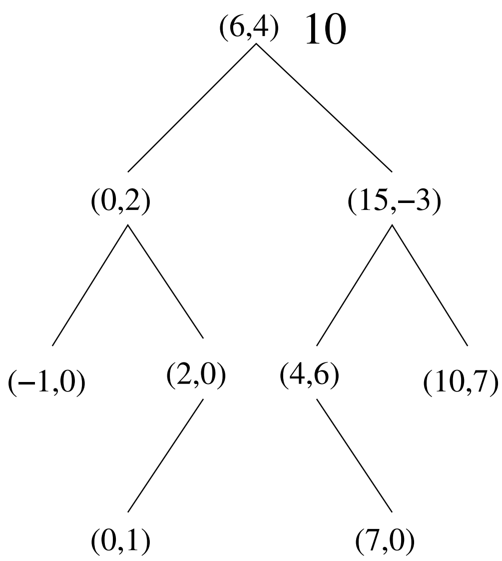


Question 7 (7 Points)

Consider a BST in which each node v contains a key as well as an additional value called *addend*. The addend of a node v is implicitly added to all keys in the subtree rooted at v. Let (*key, addend*) denote the contents of any node v. For example, the following tree contains the elements 5, 6, 7:



(a) In the following tree, write the key value of each node, e.g., the root has key value 10. (1 point)



(b) Let h be the height of a tree as defined above. Describe how to perform the following operations in $O(h)$ time:

- FIND(x,T): return YES if element x is stored in tree T.
- INSERT(x,T): inserts element x in tree T.
- PUSH(x,k,T): add k to all elements $\geq x$. (4 points)

(c) Describe how it can be insured that $h = O(\log n)$ during the above operations.
(*Hint:* Show how to perform a rotation.) (2 points)

Question 8 (4 Points)

Let $G = (V, E)$ be a bipartite graph where $V = L \uplus R$. You are given a maximum matching M in G.

(a) G' is obtained by adding an edge $e = (\ell_a, r_b)$ to G, where $\ell_a \in L$ and $r_b \in R$. Find a maximum matching in G' in $O(V + E)$ time.

(b) G' is obtained by removing an edge $e = (\ell_a, r_b) \in E$ from G. Find a maximum matching in G' in $O(V + E)$ time.

Question 9 (5 Points)

A game is played as follows. Two players alternately select distinct vertices $v_1, v_2, \ldots, v_n$ of a graph G, where, for $i > 0$, v_{i+1} is required to be adjacent to v_i. The last player able to select a vertex wins the game. Show that the first player has a winning strategy **if and only if** G has no perfect matching.

Question 10 (6 Points)

A rental company uses cars which it leases from manufacturers. The company has a requirement of cars for the next 6 months as follows:

Month	Mar.	Apr.	May	June	July	Aug.
Vehicles Required	43	41	44	39	42	45

The company can lease cars for the following costs and lengths of time: a 3-month lease for €1700, a 4-month lease for €2200, a 5-month lease for €2600. The company can undertake a lease beginning in any month. On March 1 the company has 20 cars on lease, all of which go off lease at the end of April. Formulate the problem of determining the most economical leasing policy as a mincost flow problem.

ROUGH WORK

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