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## Efficient Algorithms and Datastructures I

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### Question 1 (10 Points)

Prove the following statements:

1.  $f(n) + g(n) \in \Omega(f(n))$
2.  $f(n) \in O(g(n)) \Rightarrow f(n) + g(n) \in O(g(n))$
3.  $f(n) \in o(g(n))$  and  $g(n) \in O(h(n)) \Rightarrow h(n) \in \omega(f(n))$
4.  $f(n) \in O(g(n))$  and  $g(n) \in O(f(n)) \Leftrightarrow f(n) \in \Theta(g(n))$

State whether the following statement is true:

1.  $\frac{1}{\Omega(n)} \subseteq O(\frac{1}{n})$

### Question 2 (10 Points)

For constants  $c, \epsilon > 0$  and  $k > 1$ , arrange the following functions of  $n$  in non-decreasing asymptotic order so that  $f_i(n) = O(f_{i+1}(n))$  for two consecutive functions in your sequence. Also indicate whether  $f_i(n) = \Theta(f_{i+1}(n))$  holds or not.

$$n^k, \sqrt{n}, 2^n, n^{1+\sin(n)}, \log(n!), n^{k+\epsilon}, n^n, n, n^k(\log n)^c, n!, n \log n, 3^n, n \log \log n, n \log(n^2)$$

### Question 3 (5 Points)

Let  $f(n)$  and  $g(n)$  be asymptotically non-negative functions. Using the basic definition of  $\Theta$ -notation, prove that  $\max\{f(n), g(n)\} = \Theta(f(n) + g(n))$ .

### Question 4 (5 Points)

Show that for any real constants  $a$  and  $b$ , where  $b > 0$ ,

$$(n + a)^b = \Theta(n^b)$$