
Efficient Algorithms and Datastructures I

Question 1 (10 Points)

Solve the following recurrence relation:

$$a_n = -a_{n-1} + 9a_{n-2} - 11a_{n-3} + 4a_{n-4} \text{ with } a_0 = -7, a_1 = 4, a_2 = 48 \text{ and } a_3 = 0.$$

Question 2 (10 Points)

Calculate the value of $\sum_{i=1}^n i^2$ by setting up a recurrence relation; transforming it into a homogeneous relation via the method developed in the lecture and then solving this relation via the characteristic polynomial.

Question 3 (10 Points)

Give tight asymptotic bounds for the following recurrence relation:

$$T(n) = T(\sqrt{n}) + 1$$

Question 4 (10 Points)

Solve the following recurrence relations using generating functions:

1. $a_n = a_{n-1} + 2^{n-1}$ for $n \geq 1$ with $a_0 = 2$.
2. $a_n = 3a_{n-1} - 3a_{n-2} + a_{n-3}$ for $n \geq 3$ with $a_0 = a_1 = a_2 = 1$.