

Fundamental Algorithms 1

Note:

An R-Exercise, in this course, will refer to material that you will (or should) have discussed in a lecture earlier in your studies – on important mathematical techniques, programming issues, basic algorithms, etc. → check respective text books, online material, etc., to recapitulate these topics, if necessary!

R-Exercise 1

Prove (by induction over n) that $\frac{1}{3}n^2 + 5n + 30 \in O(n^2)$ for all $n \in \mathbb{N}^+$.

R-Exercise 2

(a) Compare the growth of the following functions using the o -, O -, and Θ -notation:

1. $n \log n$
2. n^l for all $l \in \mathbb{N}$
3. 2^n

Hint: use de l'Hôpital's rule for asymptotically larger or smaller!

(b) Try to give a simple characterization of the growth of the following expressions using the Θ -notation:

$$1) \sum_{i=1}^n \frac{1}{i} \qquad 2) \log(n!)$$

Hint for $\log(n!)$: try to prove $n^{\frac{n}{2}} \leq n! \leq n^n$ first!

R-Exercise 3

Let $l(x)$ be the number of bits of the representation of x in the binary system. Prove:

$$\sum_{i=1}^n l(i) \in \Theta(n \log n)$$

R-Exercise 4

Prove that Θ defines an equivalence relation on the set of functions $\{f \mid f: \mathbb{N} \rightarrow \mathbb{R}\}$. Use that $(f, g) \in \Theta \Leftrightarrow f \in \Theta(g)$

Note

*A Homework, in this course, is intended as a preparation of topics for the next lecture. Typically you will be asked to recapitulate (or learn) basic algorithms. In the lectures, we will **not** take the time to explain in detail how these algorithms work. Instead, we will discuss certain properties or details of the algorithm or specific implementation.*

Homework 1

Study the following basic algorithms for sorting:

InsertionSort: i.e., sort a data set by successively inserting individual items into a sorted list.

MergeSort: i.e., splitting a list into two halves, sorting the halves individually, and merging the sorted sublists \rightarrow in particular, study the **Merge** algorithm for combining two sorted lists into one.

You should understand how each algorithm proceeds to sort a given list of items.