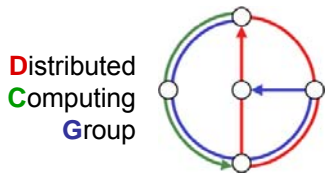


# Dynamics, Non-Cooperation, and Other Algorithmic Challenges in Peer-to-Peer Computing

Stefan Schmid

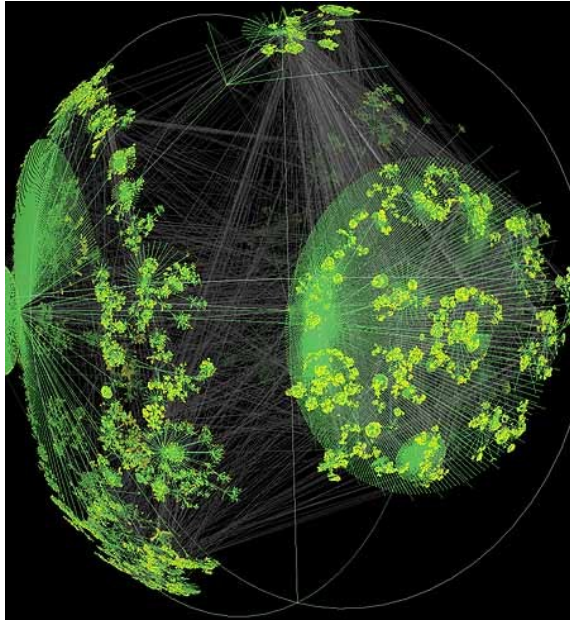


**ETH**

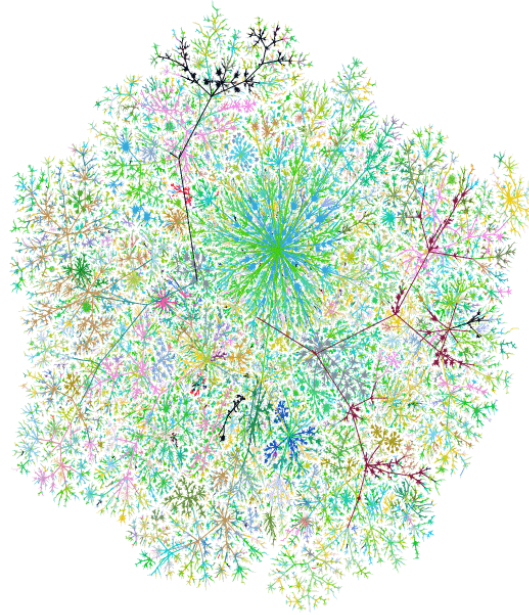
Eidgenössische Technische Hochschule Zürich  
Swiss Federal Institute of Technology Zurich

Oberseminar  
TU München, Germany  
December 2007

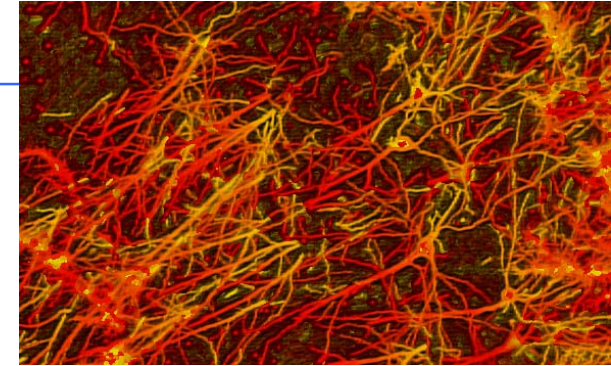
# Networks



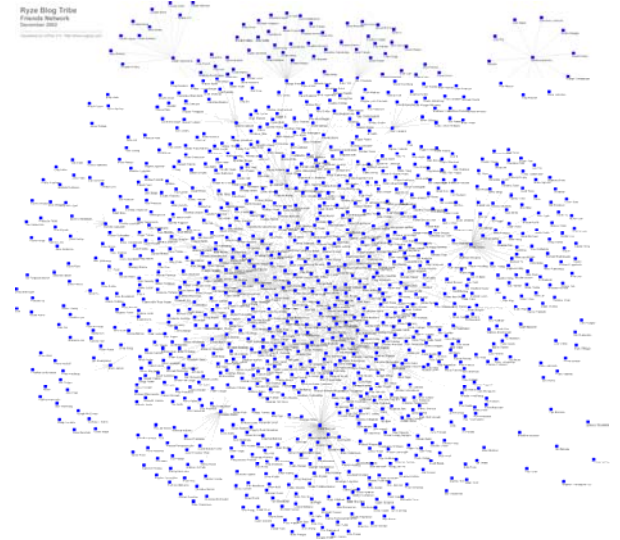
Internet Graph



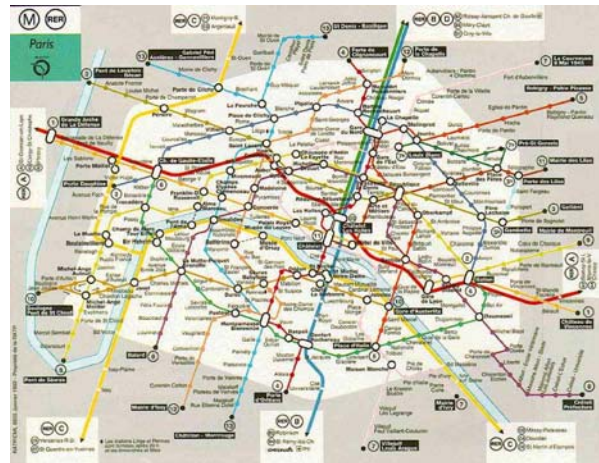
Web Graph



Neuron Networks



Social Graphs



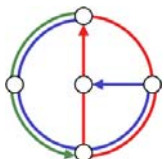
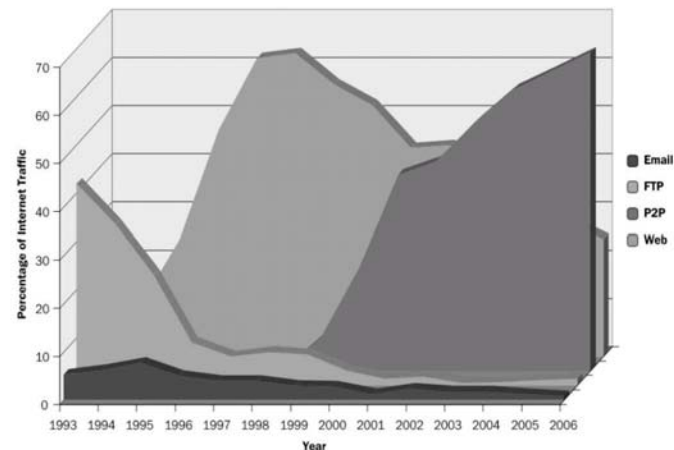
Public Transportation Networks

# This Talk: Peer-to-Peer Networks



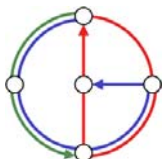
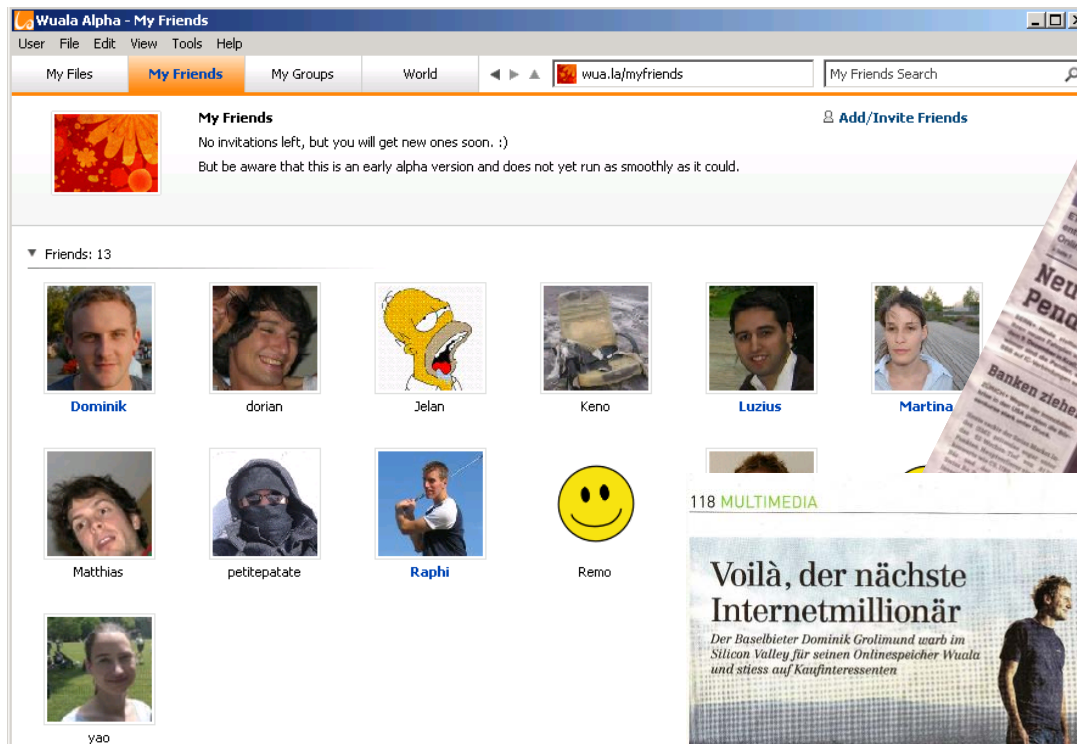
- Popular Examples:
  - BitTorrent, eMule, Kazaa, ...
  - Zattoo, Joost, ...
  - Skype, ...
  - etc.

- Important: Accounts for much **Internet traffic** today!  
(source: *cachelogic.com*)



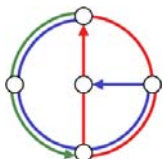
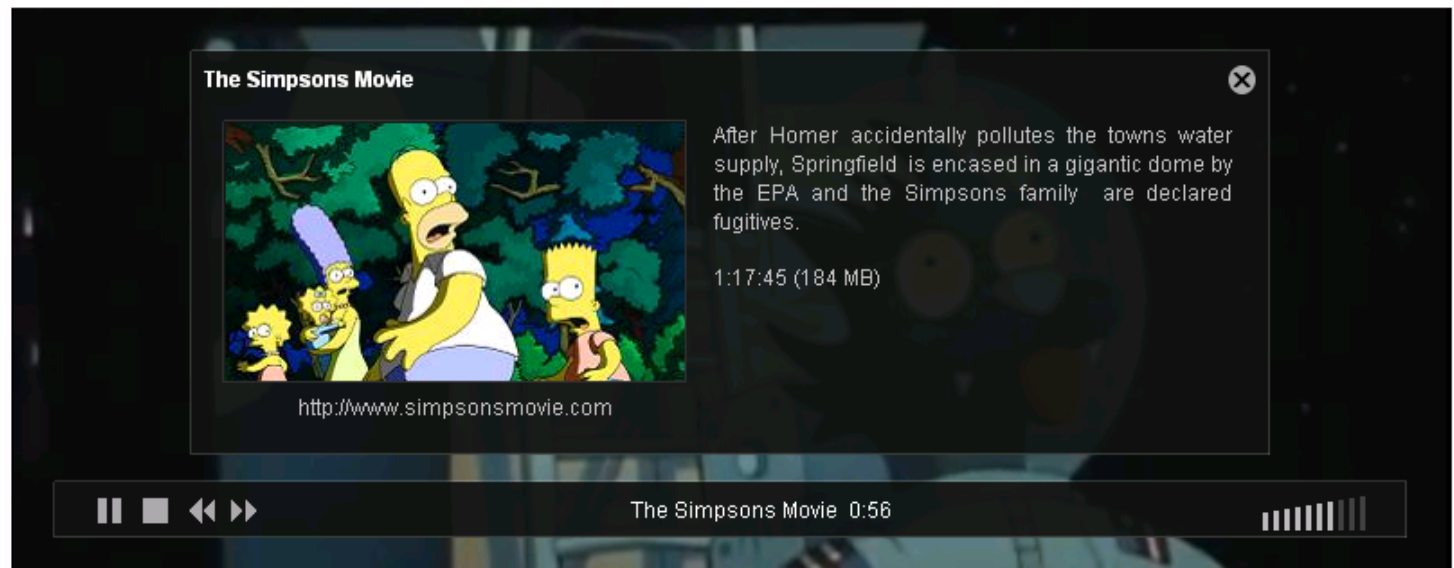
# What For?

- Many applications!
- **File sharing**, file backup, social networking: e.g. **Wuala**
  - Former student project, now **start-up** company



# What For?

- On demand and live **streaming**, e.g., **Pulsar**
  - Users / peers help to distribute contents further
  - **Cheap** infrastructure at content provider is ok!
  - Planned use: **tillate.com**, DJ event, radio, conferences, etc.

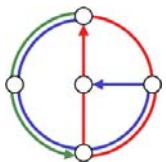


# What For?

- Peer-to-peer **games**, e.g., **xPilot**
  - Scalability (multicast updates, distributed storage, ...)
  - Cheaters? Synchronization?

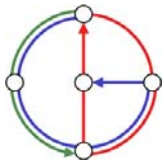
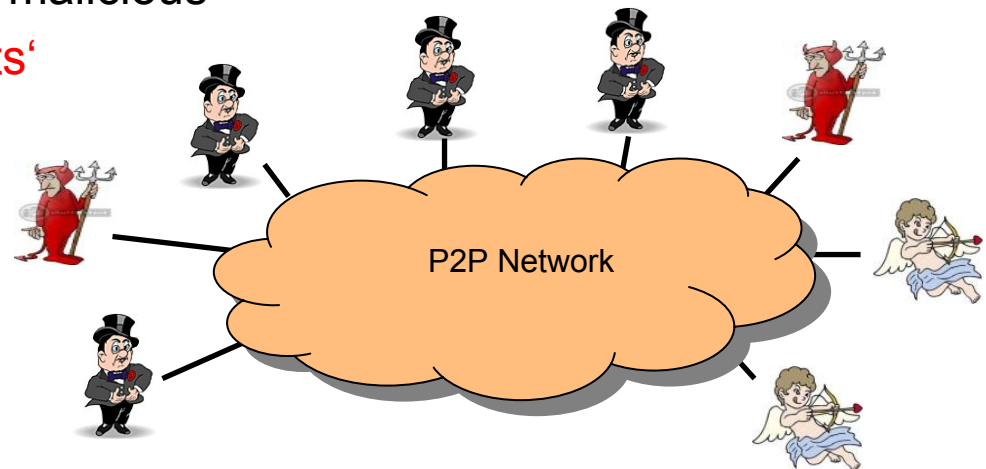
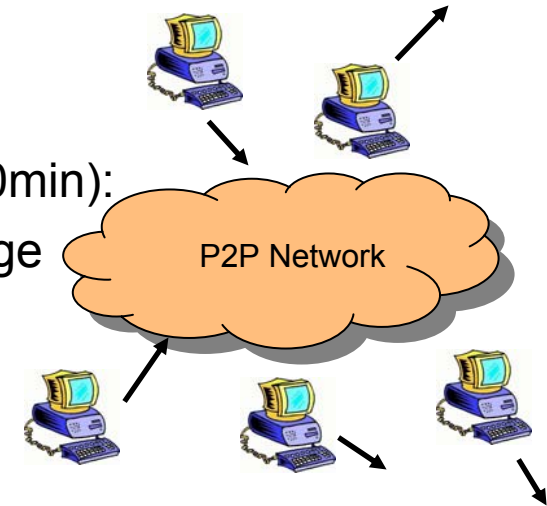


- Among many more...



# Why Are P2P Networks Interesting for Research?

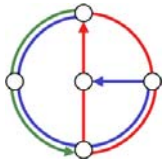
- Challenging properties...
- Peer-to-peer networks are **highly dynamic**
  - Frequent membership changes
  - If a peer only connects for downloading a file (say 60min):  
Network of 1 mio. peers implies a membership change every 3 ms on average!
  - Peers join and leave all the time and concurrently
- Participants are **humans**
  - Peers are under control of individual decision making
  - Participants may be selfish or malicious
  - Paradigm **relies on participants' contribution** of content, bandwidth, disk space, etc.!



So...



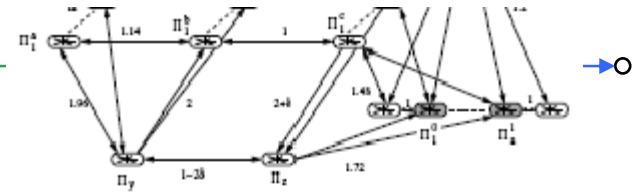
**How to provide full functionality despite dynamic, selfish and heterogeneous participants?**





# Our Research

- Often requires algorithms and theory...



$[\Lambda] \times [\mu]$  of elements  $e_1, \dots, e_k \in [\Lambda]$  with functions  $h_1, \dots, h_t$ . It suffices to show that if the functions  $h_i$  are  $\lambda$ -good for  $S$  is less than  $(1 - \delta)t/2$  functions  $h_i$  satisfy  $h$

show that the the probability of a random has

$$\mathbb{P}[h \text{ not } \lambda\text{-good for } S] < 2e^{-\lambda t/2}.$$

hash functions  $h_i, i \in \{1, \dots, t\}$  that are  $\lambda$ -good for  $S$ .

b). Using a Chernoff bound, for  $\lambda \geq 2$  we thus get

$$(1 - \delta)t(1 - 2e^{-\lambda^2/2}) < e^{-\delta^2 t(1 - 2e^{-\lambda^2/2})/2} \leq e^{-\delta^2 t}$$

is becomes less than  $1/(200KM)$ . It is also not hard to see function



ology resulting are connected as ate inter-cluster  $\hat{\pi}_x$  connects to e satisfying as- 0 and  $x_n = 1$ .

for all  $j$  that the  $(\Pi_j^a \cup \Pi_j^b \cup \Pi_j^c)$  selecting to such a

## 6.2 Satisfiable Instances

In this section, we show that if  $I$  has a satisfying assignment  $A_I$ , then there exists a Nash equilibrium in  $M_I^k$ . For this purpose, we explicitly construct a set of strategies  $s$ , which we prove to constitute a Nash equilibrium. As for notation, we define  $A_I(x_i)$  to be the assignment of  $x_i$  in  $A_I$ , i.e.,

$$A_I(x_i) := \begin{cases} 1, & x_i \text{ is set to 1 in } A_I \\ 0, & x_i \text{ is set to 0 in } A_I. \end{cases} \quad (1)$$

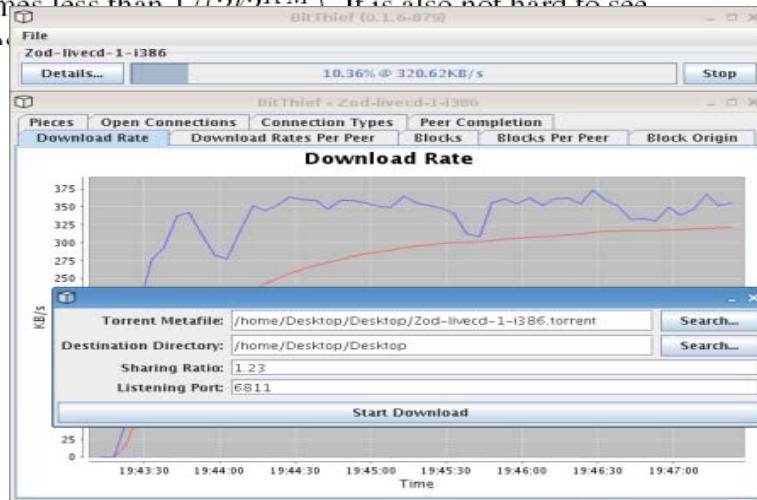
Furthermore, we define in every cluster  $\Pi_g$  a single leader peer, which we denote by  $\hat{\pi}_g$ . The role of this leader-peer is to construct all inter-cluster links going from this cluster to peers located in other clusters. The strategy of the remaining non-leader peers  $\hat{\pi}_g \in \Pi_g \setminus \{\hat{\pi}_g\}$  is to connect to the unique leader peer within their cluster. Formally, the strategy  $s_g$  for a non-leader peer  $\hat{\pi}_g \in \Pi_g \setminus \{\hat{\pi}_g\}$  is  $s_g := \{\hat{\pi}_g\}$ . For each leader-peer, we define the set of strategies  $s$  as follows:

$$s_y := \Pi_y \cup \{\hat{\pi}_x\} \cup \bigcup_{C_j \in \mathcal{C}} \{\hat{\pi}_j^a, \hat{\pi}_j^b\}$$

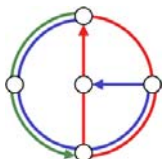
$$s_x := \Pi_x \cup \{\hat{\pi}_y\} \cup \bigcup_{z_i \in \mathcal{X}} \{\hat{\pi}_i^{A_I(x_i)}\}$$

$$s_c := \Pi_c \cup \bigcup_{z_i \in \mathcal{X}} \{\hat{\pi}_i^a \cup \hat{\pi}_i^b\} \cup \bigcup_{C_j \in \mathcal{C}} \{\hat{\pi}_j^a \cup \hat{\pi}_j^b \cup \hat{\pi}_j^c\}$$

$$s_j^a := \Pi_j^a \cup \{\hat{\pi}_c, \pi_y, \hat{\pi}_j^b\}, \quad \forall C_j \in \mathcal{C}$$



2007



# Outline of Talk

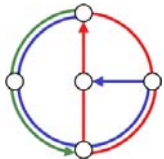


- Congestion Control (IPTPS 2006, IWQoS 2006)
- BitThief: Today's congestion control can be exploited by selfish participants (HotNets 2006)
- Game-theoretic analysis of selfish congestion control (IPTPS 2006, PODC 2006)

very briefly ...

very briefly ...

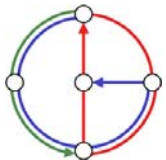
more in detail ...



# Outline of Talk



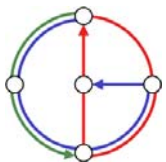
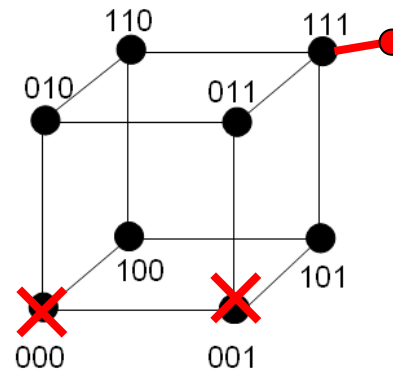
- **Coping with churn (IPTPS 2005, IWQoS 2006)**
- BitThief: Today's system can be exploited by selfish participants (HotNets 2006)
- Game-theoretic analysis of selfish behavior (IPTPS 2006, PODC 2006)



# High Dynamics on Hypercube?



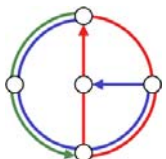
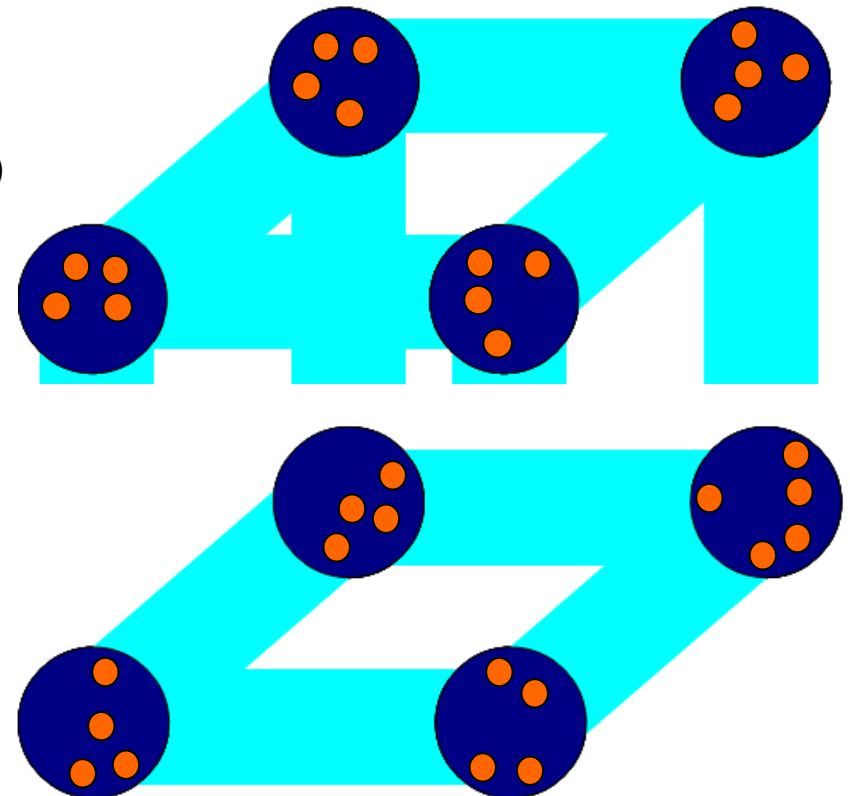
- Motivation: Why is dynamics a problem?
- Frequent membership changes are called **churn**
- How to maintain **low network diameter** and **low node degree** in spite of dynamics? How to prevent data loss?
- Popular topology: Hypercube
  - Logarithmic diameter, logarithmic node degree



# Resilient Solution



- Simulating the hypercube!
  - Several peers „simulate“ one node
- Maintenance algorithm:
  - Distribute peers evenly among IDs (nodes)  
(-> **token distribution** problem)
  - Distributed estimation of total number of peers and **adapt dimension** of hypercube when necessary
- Thus, at least one peer per ID (node) at any time!

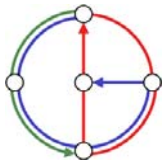


# Analysis



Even if an adversary adds and removes a **logarithmic** number of peers per communication round in a worst-case manner, the network diameter is always **logarithmic** and **no data is lost**.

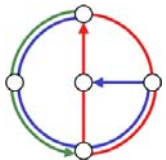
- Also works for other topologies, e.g., pancake graph!



# Outline of Talk



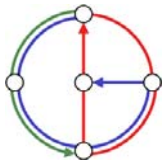
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- Game-theoretic analysis of selfish behavior (IPTPS 2006, PODC 2006)



# Outline of Talk



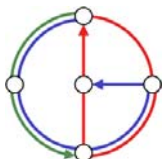
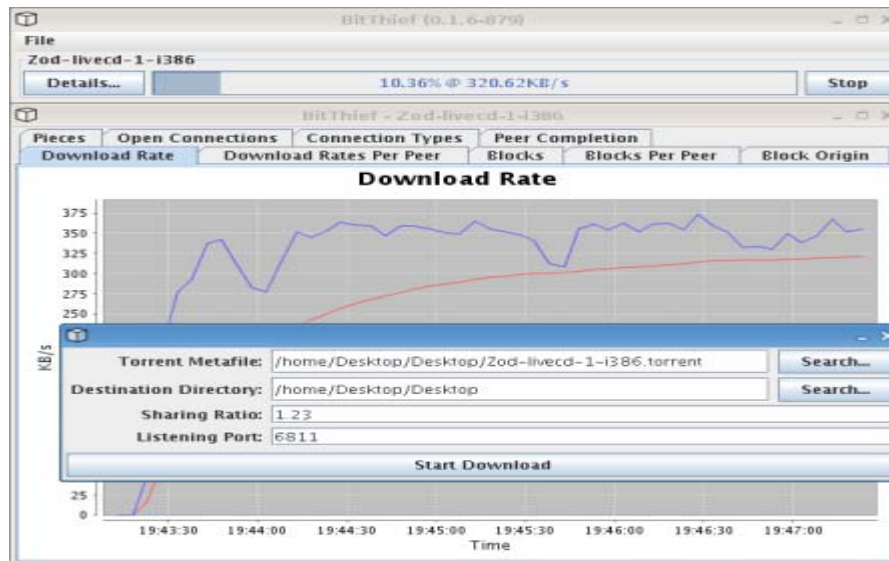
- Coping with churn (IPTPS 2005, IWQoS 2006)
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- Game-theoretic analysis of selfish behavior (IPTPS 2006, PODC 2006)





# BitThief

- Case Study: Free riding in BitTorrent
- BitThief: Free-riding BitTorrent client
  - written in Java
  - Downloads entire files efficiently **without uploading** any data
  - Despite BitTorrent's **Tit-for-Tat** incentive mechanism!

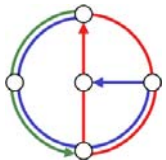
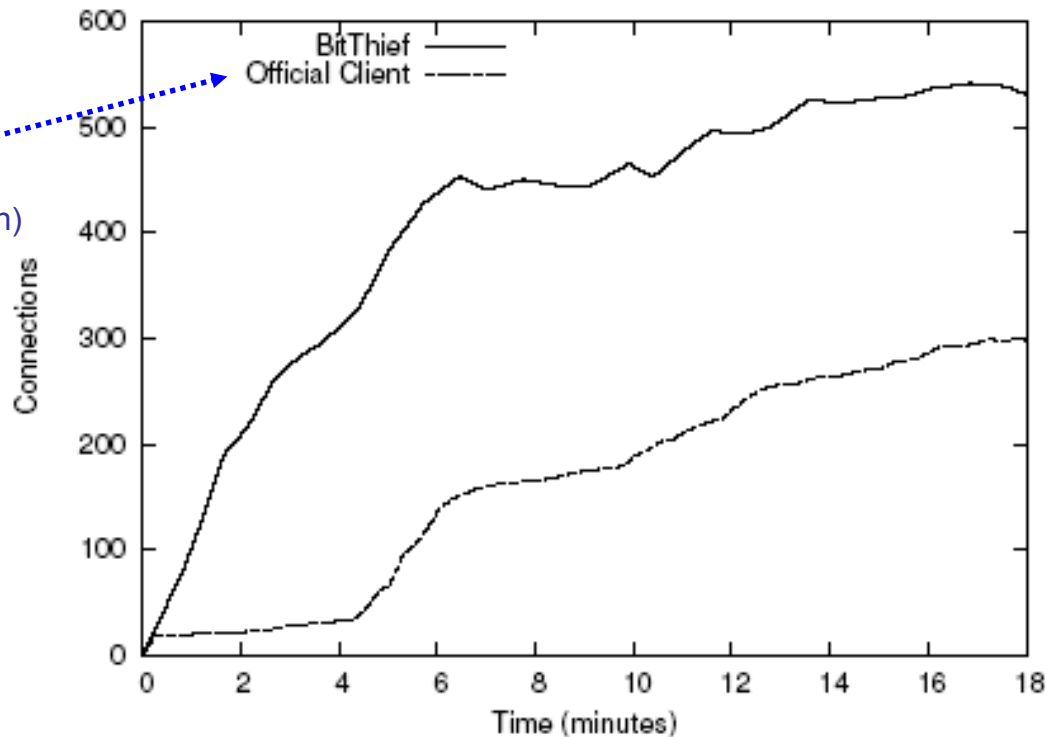


# BitThief's Exploits (1)



- Exploit 1: Exploit **unchoking** mechanism
  - New peer has nothing to offer -> BitTorrent peers have unchoking slots
  - Exploit: Open as **many TCP connections** as possible!

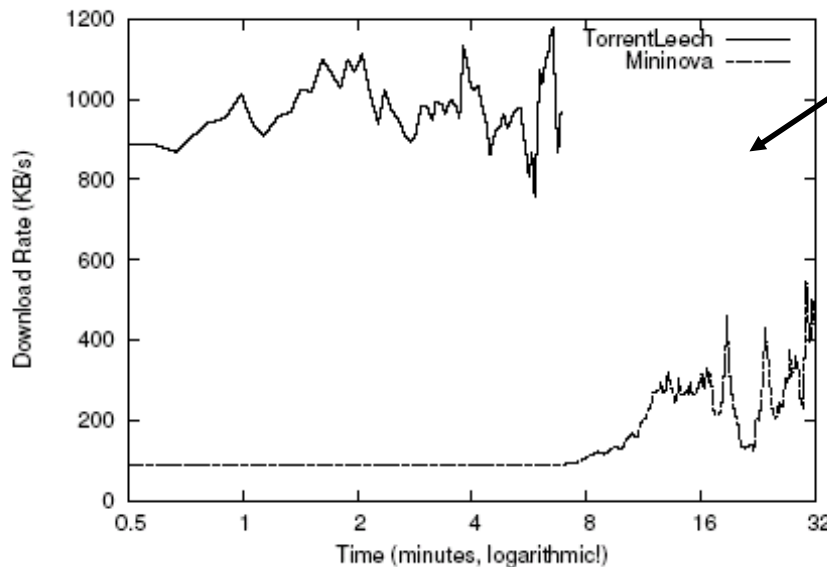
V4.20.2 from  
bittorrent.com  
(written in Python)



# BitThief's Exploits (2)

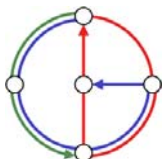


- Exploit 2: Sharing Communities
  - Communities require user registration and ban uncooperative peers
  - Many seeders! (= peers which only upload)
  - Exploit: **Fake tracker announcements**, i.e., report large amounts of uploaded data



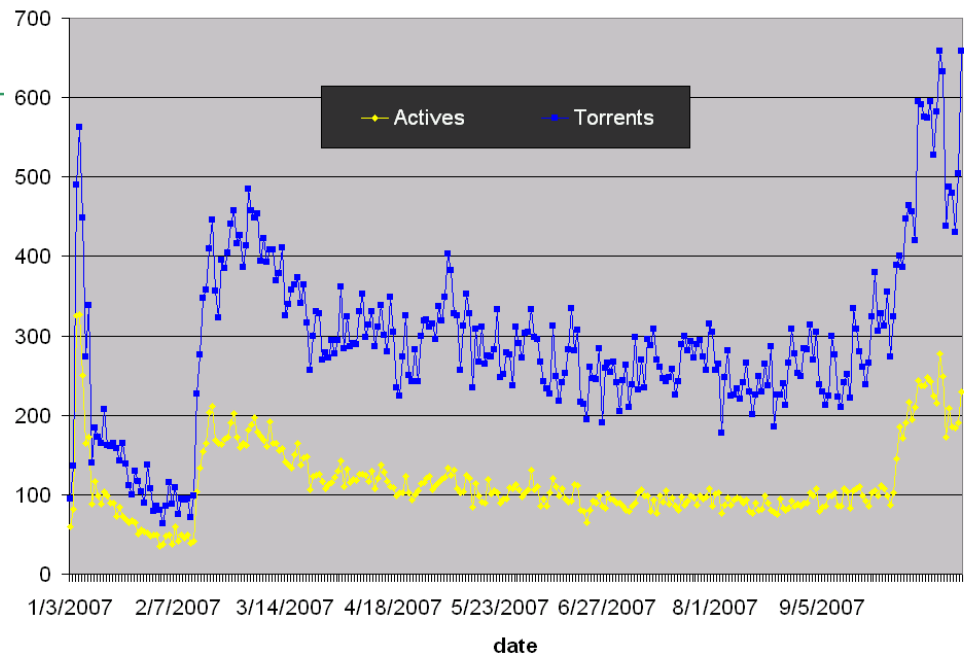
4 x faster!

(BitThief had a faked sharing ratio of 1.4; in both networks, BitThief connected to roughly 300 peers)



# Some Reactions

- Selfishness in p2p computing seems to be an important topic – inside and outside academic world: **blogs**, emails, up to 100 paper downloads per day! (>3000 in January 2007)
- Recommendation on **Mininova FAQ** (!)
- But still some concerns...



"Anyhow, bitthief is a client which I've been waiting for so long, I mean.. bitcomet bent the rules but never really broke any of them.. that much Bitthief is an interesting client in that it openly says "fuck you, and fuck your swarm" to the torrent community. I wonder how fast this will get banned at every tracker alive. As others have said, this makes bittorrent look like a sunday school boy."

-----Original Message-----

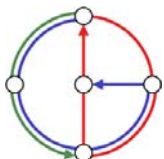
From: Warren Henning [mailto:warren.henning@gmail.com]  
Sent: Friday, January 12, 2007 3:03 PM  
To: [lochert@tik.ee.ethz.ch](mailto:lochert@tik.ee.ethz.ch); [schmiste@tik.ee.ethz.ch](mailto:schmiste@tik.ee.ethz.ch); [wattenhofer@tik.ee.ethz.ch](mailto:wattenhofer@tik.ee.ethz.ch)  
Subject: Stop distributing BitThief, you jerks!

BitTorrent is a beautiful thing and you are intentionally fucking it up by distributing software that is apparently specifically designed to attack the entire basis of the function of BitTorrent, software that serves no legitimate purpose.

Luckily it apparently requires having a JRE installed right now, and the knuckle-dragging numbskulls you've worked so hard to cater to are probably too lazy to install that.

You people piss me off.

Warren Henning

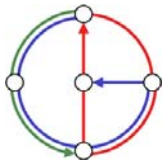


# Effects of Selfishness?



- Question remains:
- Is selfishness really a problem in p2p networks?
  - Tools to estimate impact of selfishness: **game theory!**

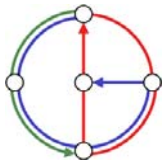
Tackled next!



# Outline of Talk



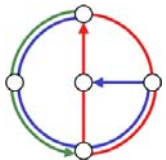
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- BitThief: Today's system can be exploited by selfish participants (HotNets 2006)
- Game-theoretic analysis of selfish behavior (IPTPS 2006, PODC 2006)



# Outline of Talk



- Coping with churn (IPTPS 2005, IWQoS 2006)
- BitThief: Today's system can be exploited by selfish participants (HotNets 2006)
- **Game-theoretic analysis of selfish behavior (IPTPS 2006, PODC 2006)**



# Selfishness in P2P Networks

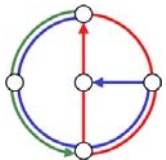
- How to study the impact of non-cooperation / selfish behavior?
- Example: Impact of **selfish neighbor selection** in **unstructured P2P systems**
- Goals of selfish peer:
  - It wants to have small latencies, **quick look-ups**
  - It wants to have **small set of neighbors** (maintenance overhead)



What is the impact on the P2P topologies?

Efficiency

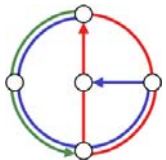
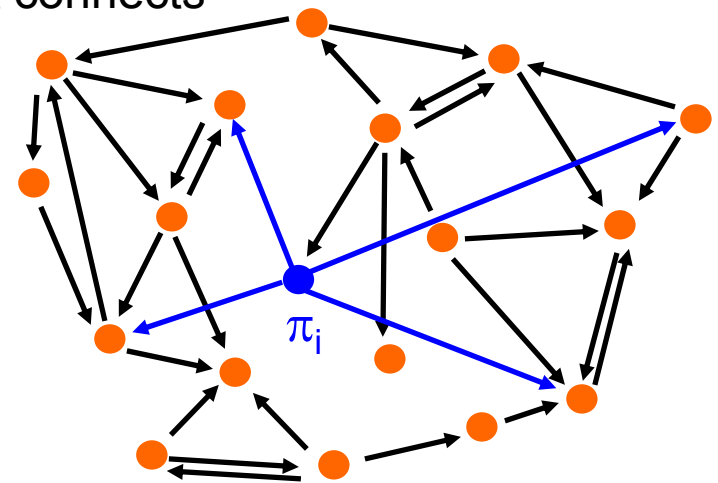
Stability





# Model – The “Locality Game”

- Model inspired by **network creation game** [Fabrikant et al, PODC'03]
  - Sparked much future research, e.g., study of **bilateral links** (both players pay for link) rather than unilateral by Corbo & Parkes at PODC'05
- $n$  peers  $\{\pi_0, \dots, \pi_{n-1}\}$  distributed in a **metric space**
  - defines distances ( $\rightarrow$  latencies) between peers
  - triangle inequality holds
  - Examples: Euclidean space, doubling or growth-bounded metrics, 1D line,...
- Each peer can choose to which other peer(s) it connects
- Yields a **directed graph**...



# Model – The “Locality Game”



- Goal of a selfish peer:

- Only **little memory** used
- Small **maintenance** overhead

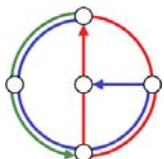
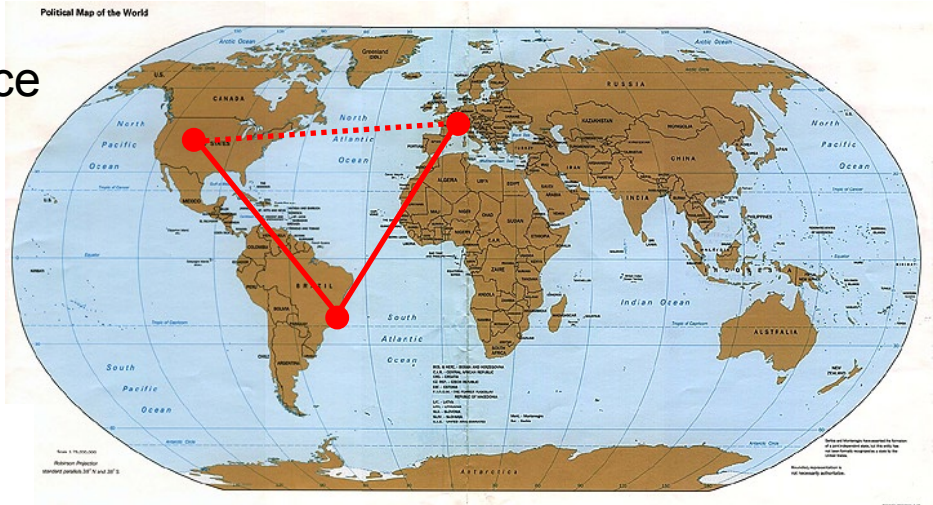
- (1) Maintain a small number of neighbors only (**out-degree**)
- (2) Small **stretches** to all other peers in the system

## Fast lookups!

- Shortest path using links in G...
- ... divided by shortest direct distance

**LOCALITY!**

**Classic P2P trade-off!**



# Model – The “Locality Game”

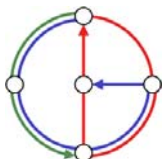


- Cost of a peer  $\pi_i$ :
  - Number of neighbors (**out-degree**) times a **parameter  $\alpha$**
  - plus **stretches** to all other peers
  - $\alpha$  captures the trade-off between link and stretch cost

$$cost_i = \alpha \cdot outdeg_i + \sum_{i \neq j} stretch_G(\pi_i, \pi_j)$$

- Goal of a peer: **Minimize its cost!**

- $\alpha$  is cost per link
- $>0$ , otherwise solution is a complete graph



# Model – Social Cost

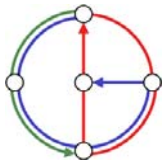


- **Social Cost** is the sum of costs of individual peers
- System designer wants small social costs (-> efficient system)
- **Social Optimum (OPT)**
  - Topology with minimal social cost of a given problem instance
  - “**topology formed by collaborating peers**”!



- What topologies do *selfish peers* form?

→ Concepts of **Nash equilibrium** and **Price of Anarchy**



# Model – Price of Anarchy



- **Nash equilibrium**

- “Result” of selfish behavior → “**topology formed by selfish peers**”
- Network where no peer can reduce its costs by changing its neighbor set given that neighbor sets of the other peers remain the same

- **Price of Anarchy**

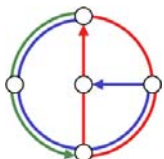
- Captures the **impact of selfish behavior** by comparison with optimal solution: ratio of social costs



**What is the Price of Anarchy of our “Locality Game”?**

**Is there actually a Nash equilibrium...?**

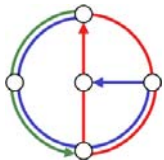
$$PoA := \max_I \frac{NASH(I)}{OPT(I)}$$



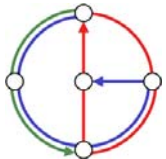
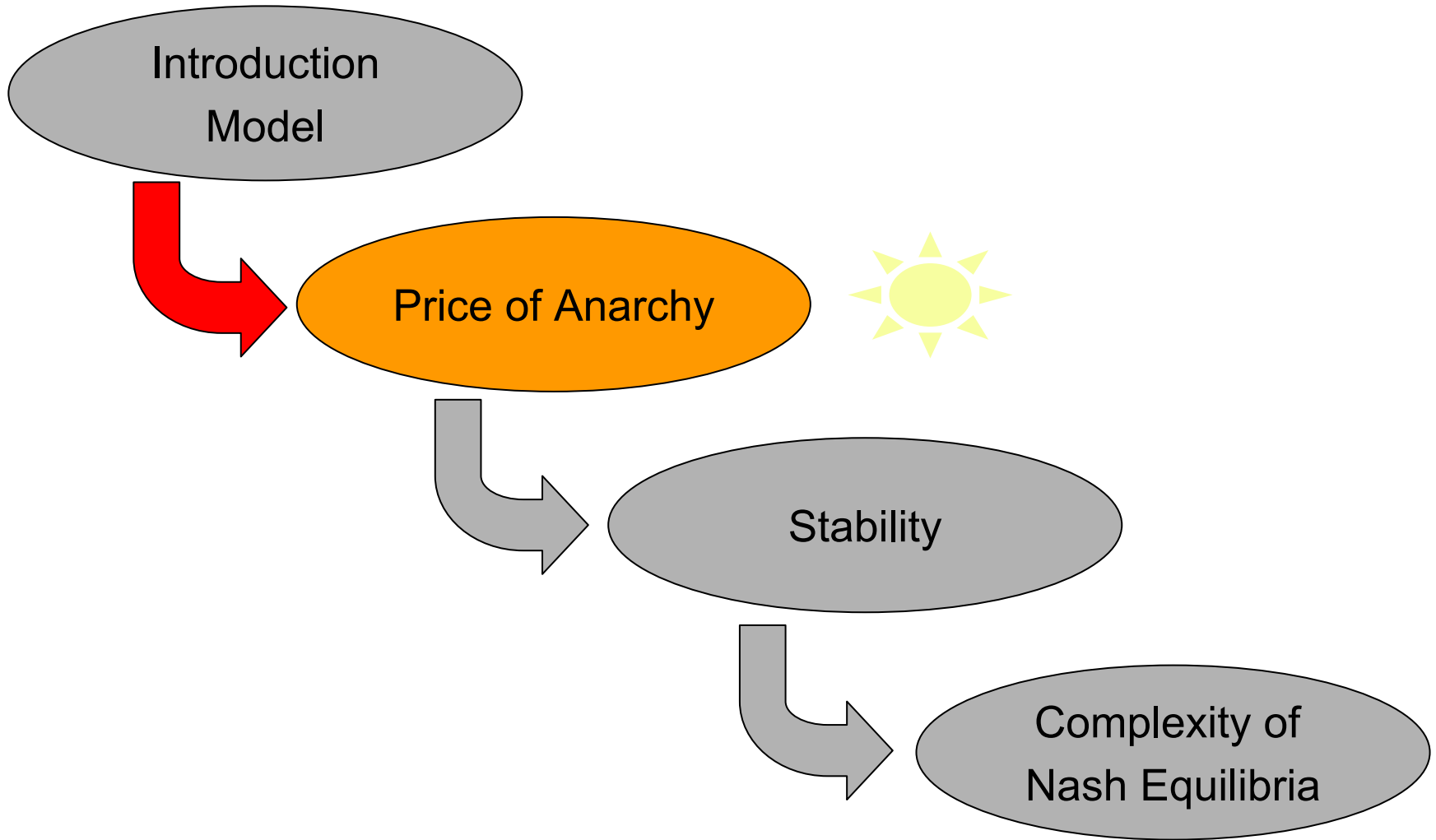
# Related Work



- The “*Locality Game*” is inspired by the “*Network Creation Game*”
- Differences:
  - In the Locality Game, nodes are located in a **metric space**
  - Definition of stretch is based on metric-distance, not on hops!
  - The Locality Game considers **directed links**
  - Yields new optimization function



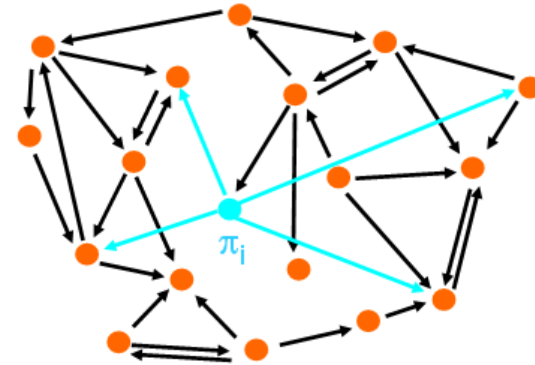
# Overview



# Analysis: Lower Bound for Social Optimum?

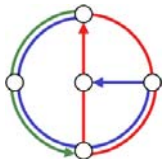


$$cost_i = \alpha \cdot outdeg_i + \sum_{i \neq j} stretch_G(\pi_i, \pi_j)$$



- Compute upper bound for PoA => need lower bound for social opt and an upper bound on Nash equilibrium cost
- OPT > ?
  - Sum of all the peers' individual costs must be at least?
  - Total link costs > ? (Hint: directed connectivity)
  - Total stretch costs > ?

Your turn! 😊

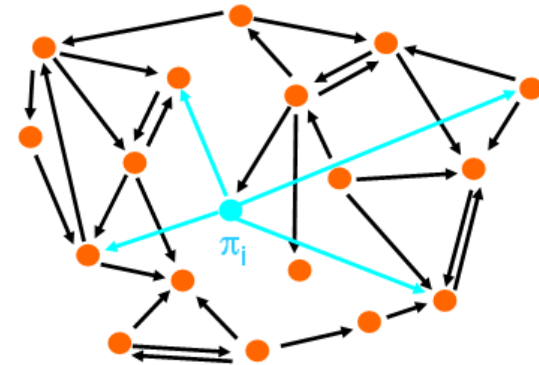




# Analysis: Social Optimum



- For connectivity, **at least  $n$  links** are necessary  
 →  $OPT \geq \alpha n$
- Each peer has **at least stretch 1** to all other peers  
 →  $OPT \geq n \cdot (n-1) \cdot 1 = \Omega(n^2)$



**$OPT \in \Omega(\alpha n + n^2)$**

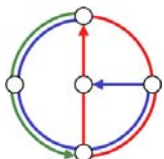
- Now: Upper Bound for NE? In any Nash equilibrium, **no stretch exceeds  $\alpha+1$** : total stretch cost at most  $O(\alpha n^2)$   
 → otherwise it's worth connecting to the corresponding peer (stretch becomes 1, edge costs  $\alpha$ )
- Total link cost also at most  $O(\alpha n^2)$

**$NASH \in O(\alpha n^2)$**

Really...?

Can be bad for large  $\alpha$

**$Price\ of\ Anarchy \in O(\min\{\alpha, n\})$**



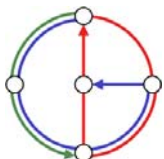
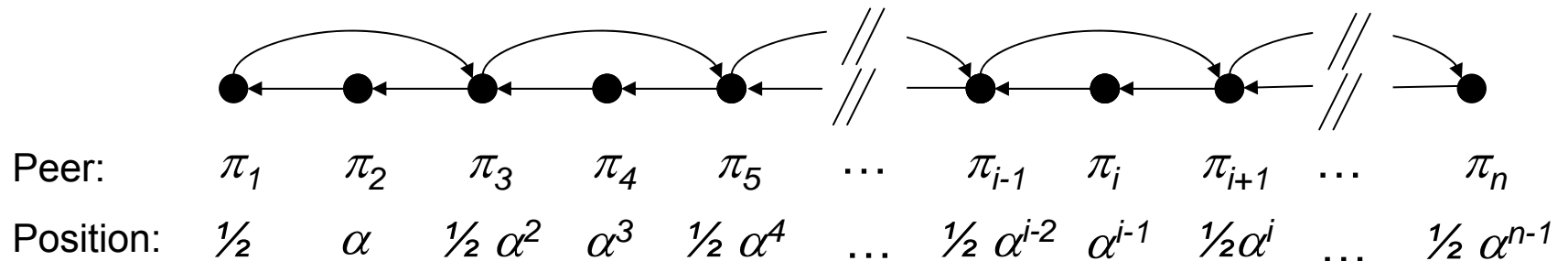
# Analysis: Price of Anarchy (Lower Bound)



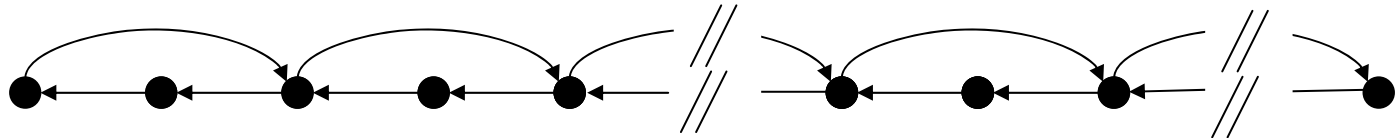
- Price of anarchy is **tight**, i.e., it also holds that

**The Price of Anarchy is  $\text{PoA} \in \Omega(\min\{\alpha, n\})$**

- This is already true in a **1-dimensional Euclidean space**:



# Analysis: Price of Anarchy (Lower Bound)



Peer:	$\pi_1$	$\pi_2$	$\pi_3$	$\pi_4$	$\pi_5$	...	$\pi_{i-1}$	$\pi_i$	$\pi_{i+1}$	...	$\pi_n$
Position:	$1/2$	$\alpha$	$1/2 \alpha^2$	$\alpha^3$	$1/2 \alpha^4$	...	$1/2 \alpha^{i-2}$	$\alpha^{i-1}$	$1/2 \alpha^i$	...	$1/2 \alpha^{n-1}$

To prove:

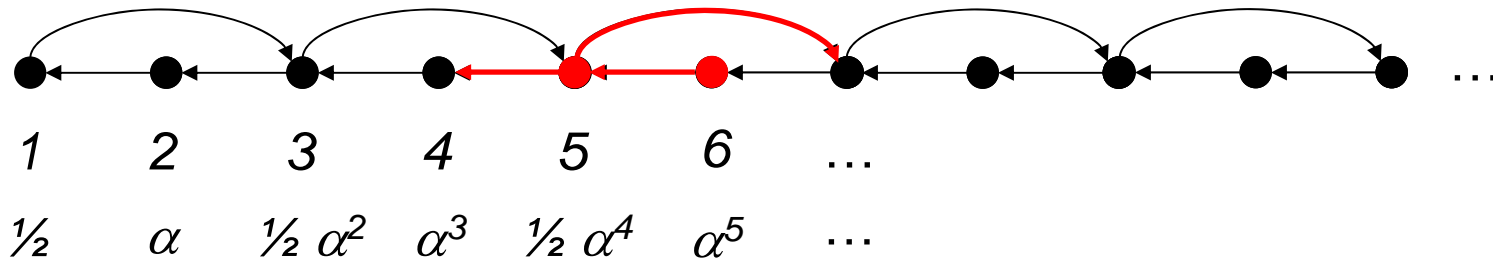
- (1) “is a selfish topology” = instance forms a **Nash equilibrium**
- (2) “has large costs compared to OPT”  
= the **social cost** of this Nash equilibrium is  $\Theta(\alpha n^2)$

Note: **Social optimum** is at most  $O(\alpha n + n^2)$ :

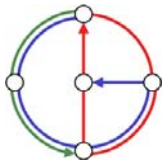


$O(n)$  links of cost  $\alpha$ , and all stretches = 1

# Analysis: Topology is Nash Equilibrium



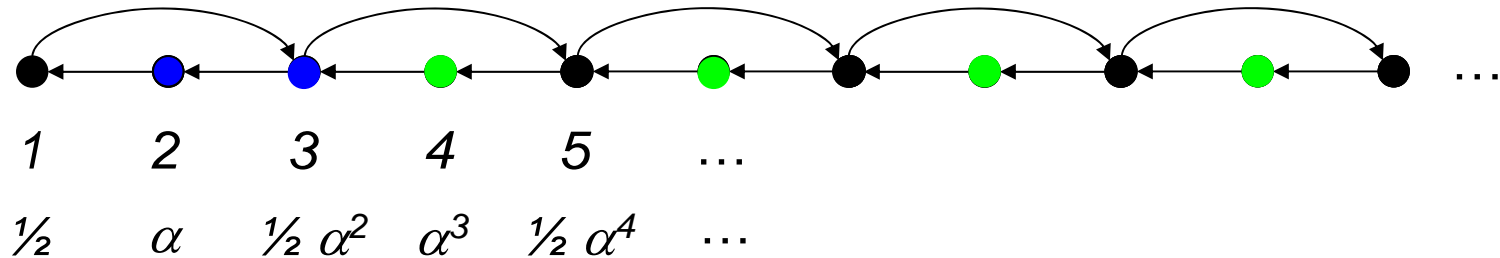
- Proof Sketch: Nash?
  - Even peers:
    - For **connectivity**, at least **one link to a peer on the left** is needed (cannot change neighbors without increasing costs!)
    - With this link, all peers on the left can be reached with an **optimal stretch 1**
    - **Links to the right cannot reduce the stretch costs** to other peers by more than  $\alpha$
  - Odd peers:
    - For **connectivity**, at least **one link to a peer on the left** is needed
    - With this link, all peers on the left can be reached with an **optimal stretch 1**
    - Moreover, it can be shown that **all alternative or additional links** to the right entail larger costs



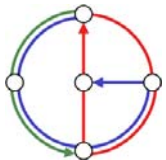
# Analysis: Topology has Large Costs



- Idea why social cost are  $\Theta(\alpha n^2)$ :  $\Theta(n^2)$  stretches of size  $\Theta(\alpha)$



- The stretches from all **odd peers  $i$**  to a **even peers  $j > i$**  have stretch  $> \alpha/2$
- And also the stretches between **even peer  $i$**  and **even peer  $j > i$**  are  $> \alpha/2$



# Analysis: Price of Anarchy (Lower Bound)



- Price of anarchy is **tight**, i.e., it holds that

**The Price of Anarchy is  $\text{PoA} \in \Theta(\min\{\alpha, n\})$**

- This is already true in a **1-dimensional Euclidean space**
- Discussion:

Need no incentive mechanism

→ For small  $\alpha$ , the Price of Anarchy is small!

Need an incentive mechanism

→ For large  $\alpha$ , the Price of Anarchy grows with  $n$ !

*Example: Network with many small queries / files -> latency matters,  $\alpha$  large, selfishness can deteriorate performance!*

# What about stability...?



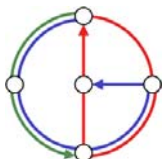
- We have seen:

**Unstructured p2p topologies may deteriorate due to selfishness!**

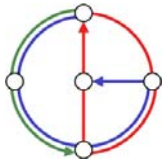
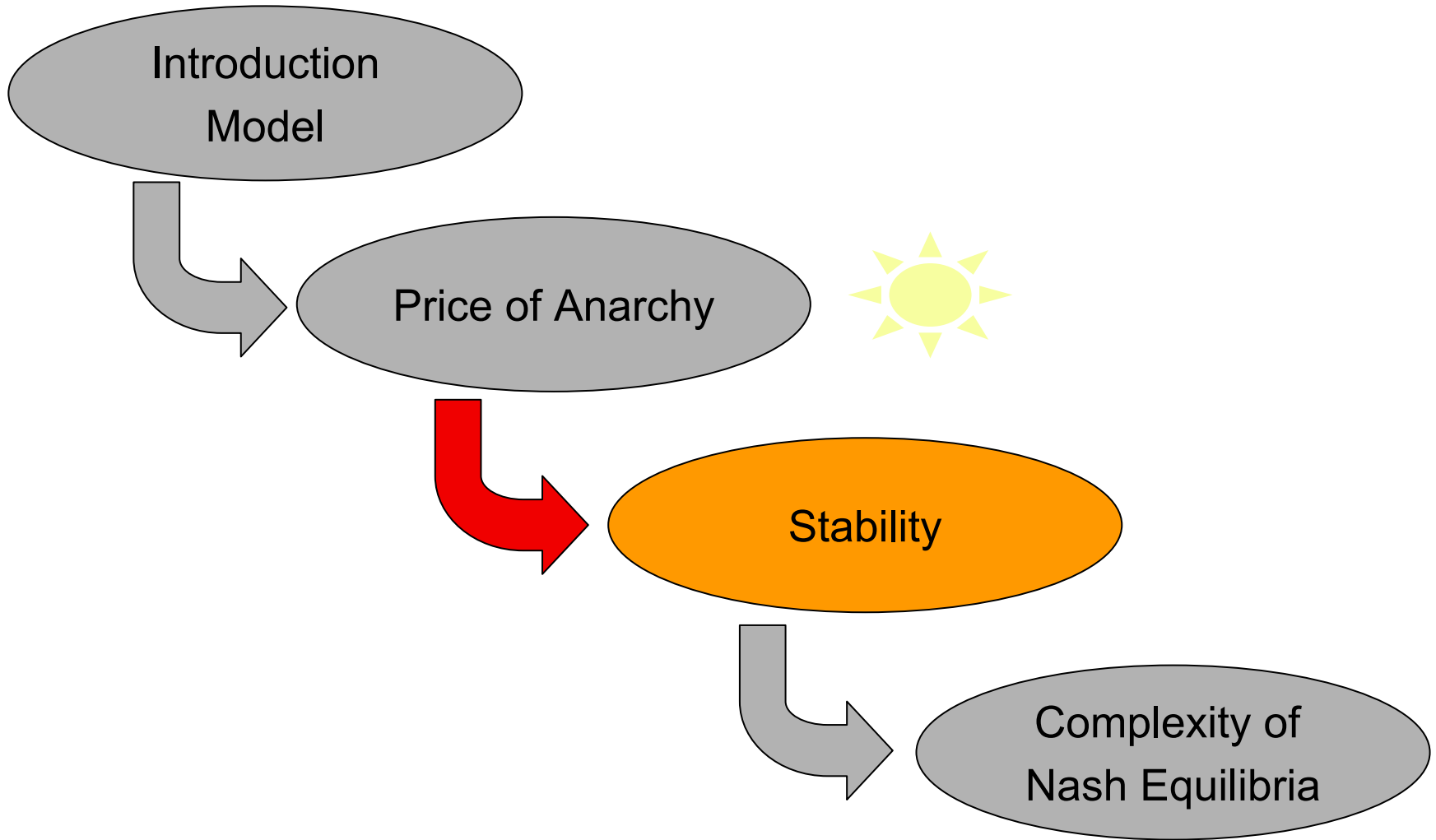
- What about other effects of selfishness...?
- ... selfishness can cause even more harm...!



**Even in the absence of churn, mobility or other sources of dynamism, the system may never stabilize (i.e., P2P system may never reach a Nash equilibrium)!**



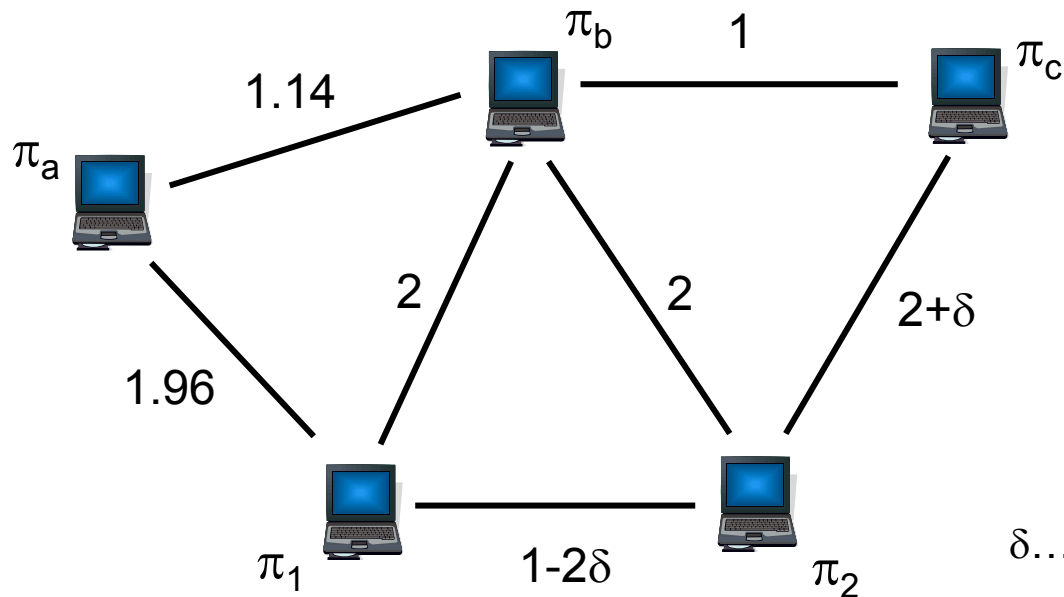
# Overview



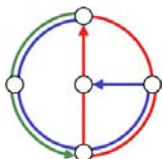


# What about stability...?

- Consider the following simple toy-example
- Let  $\alpha=0.6$  (for illustration only!)
- 5 peers in **Euclidean** plane as shown below (other distances implicit)
- What topology do they form...?



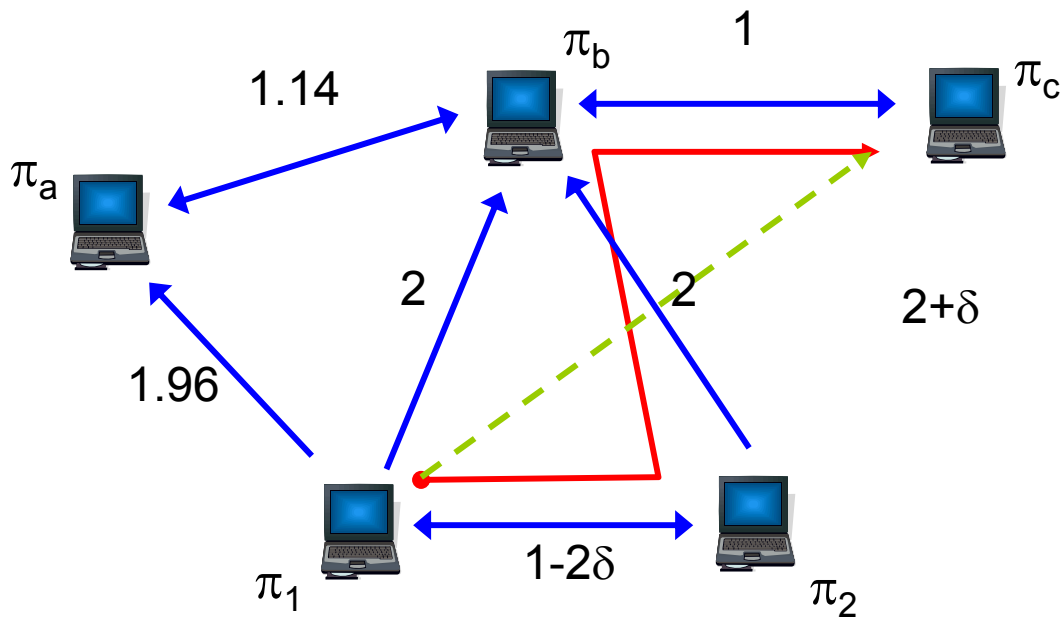
$\delta$ ...arbitrary small number



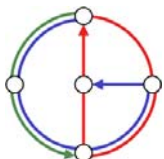
# What about stability...?



- Example sequence:
  - Bidirectional links shown must **exist in any NE**, and peers at the bottom must have directed links to the upper peers somehow: considered now! (ignoring other links)



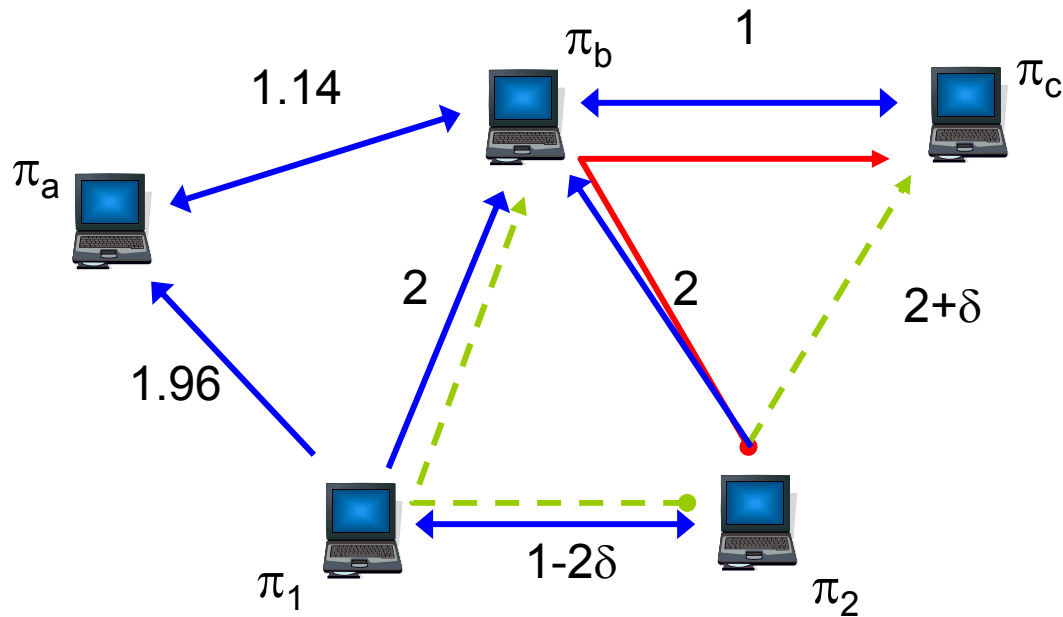
$$\underbrace{\frac{1 - 2\delta + 2 + 1}{d(\pi_1, \pi_c)}}_{\text{stretch}(\pi_1, \pi_c)} + \underbrace{\frac{1 - 2\delta + 2}{2}}_{\text{stretch}(\pi_1, \pi_b)} > \alpha + 1 + \underbrace{\frac{3}{d(\pi_1, \pi_c)}}_{\text{stretch}(\pi_1, \pi_c)}$$



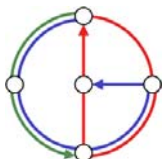
# What about stability...?



- Example sequence:



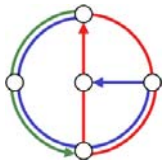
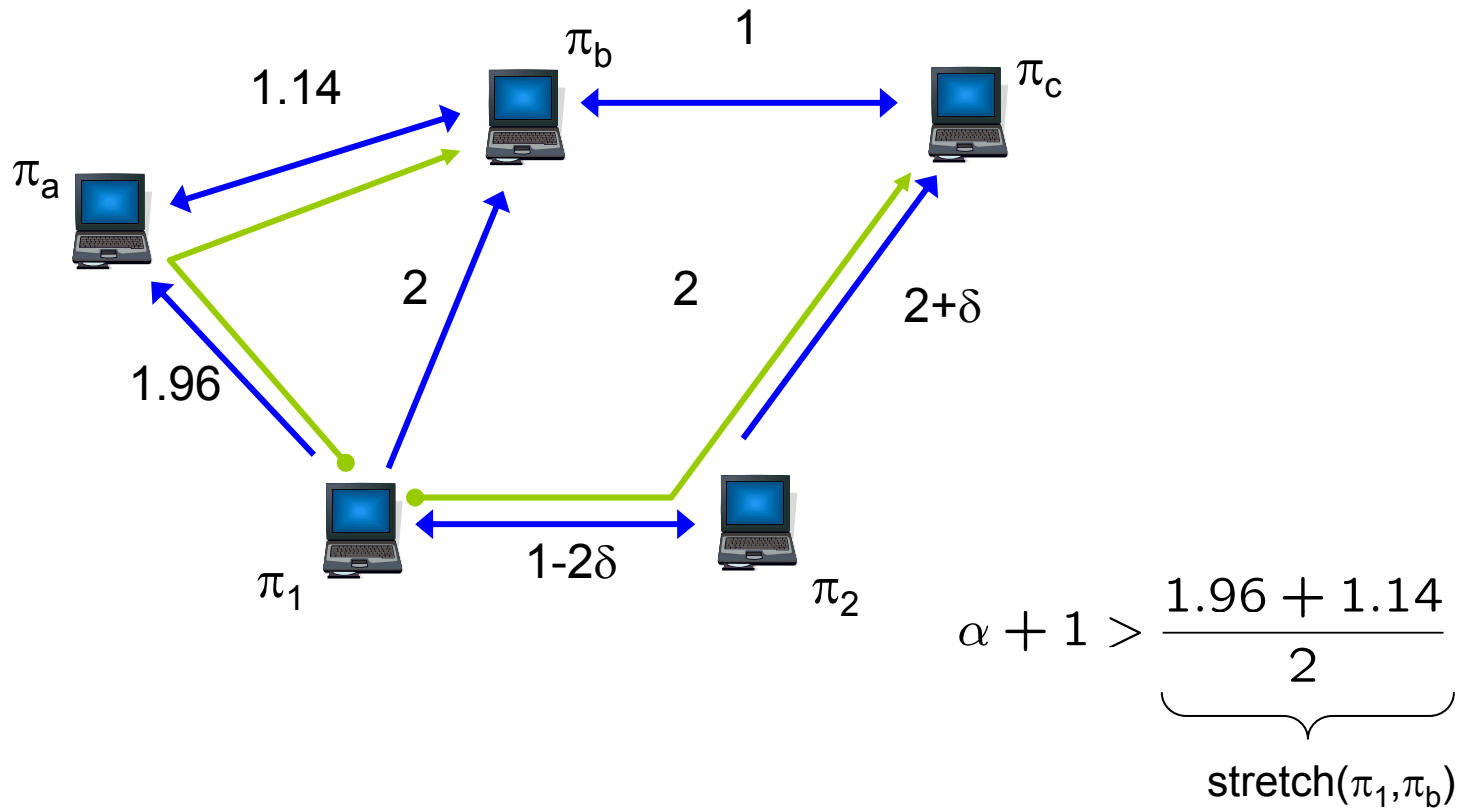
$$1 + \underbrace{\frac{1+2}{2+\delta}}_{\text{stretch}(\pi_2, \pi_c)} > \underbrace{\frac{1-2\delta+2}{2}}_{\text{stretch}(\pi_2, \pi_b)} + 1$$



# What about stability...?



- Example sequence:



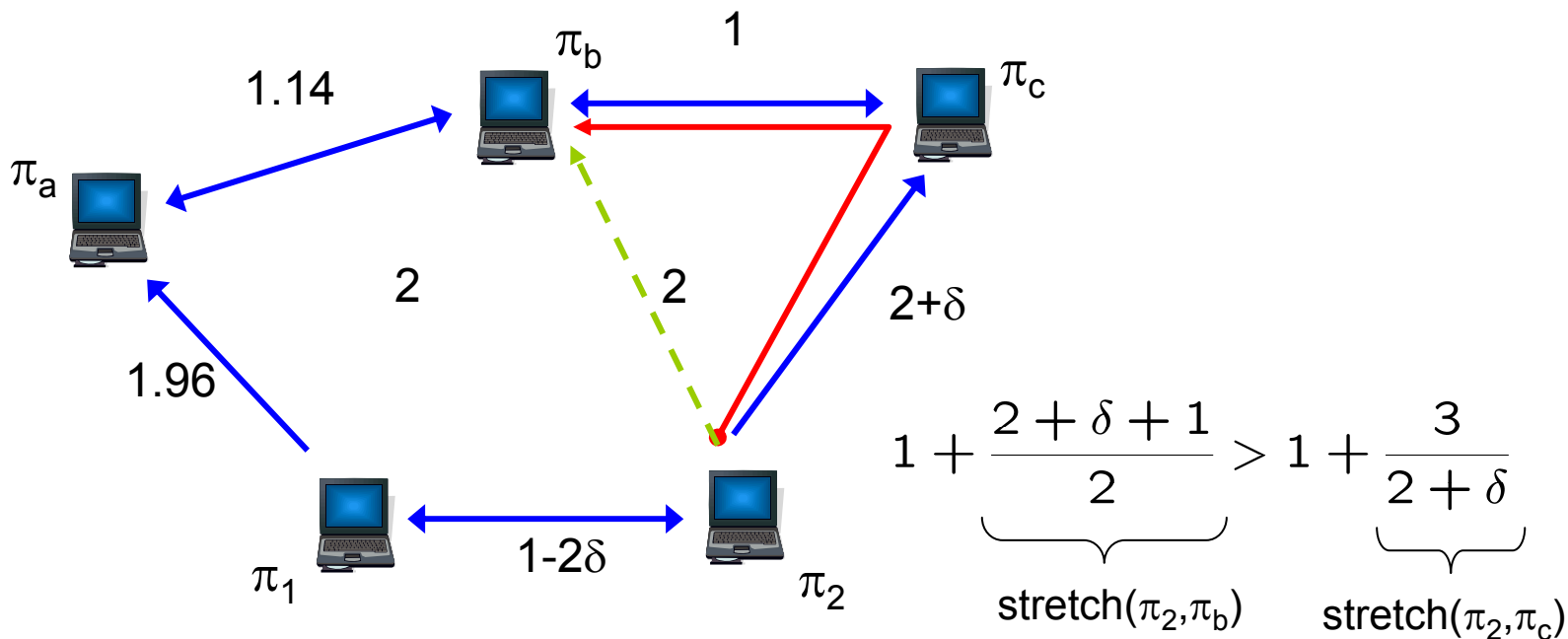
# What about stability...?



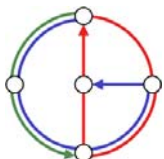
- Example sequence:

Again initial situation

→ Changes **repeat forever!**

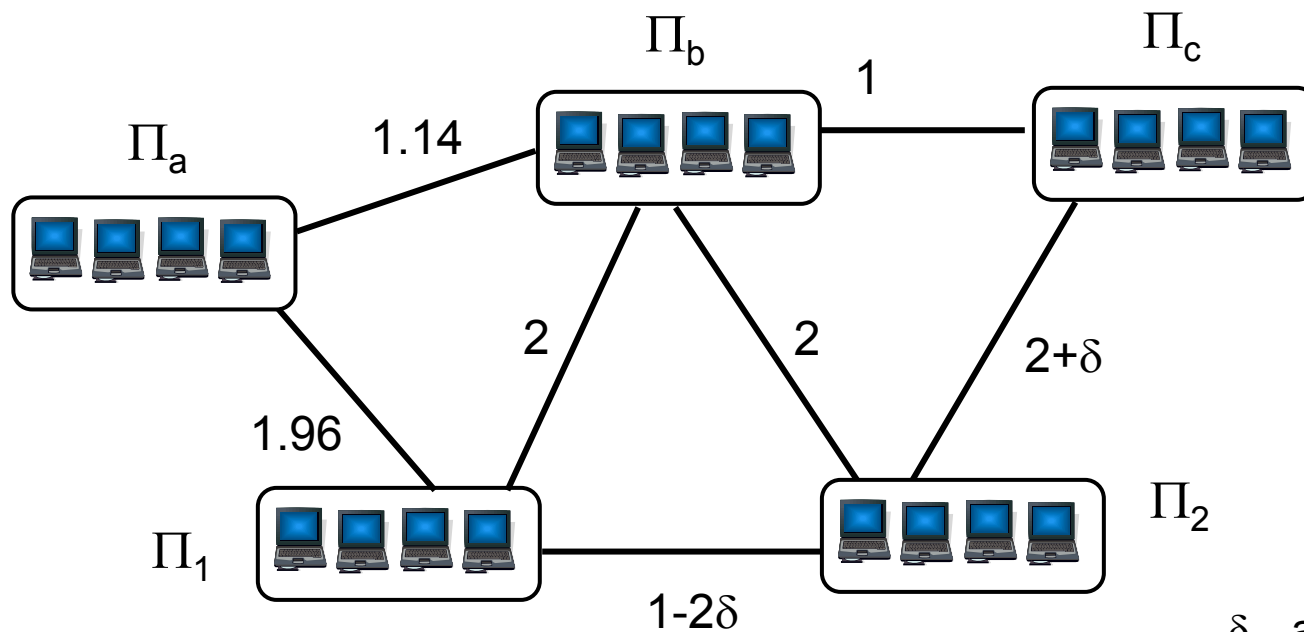


Generally, it can be shown that **for all  $\alpha$** , there are networks, that do not have a Nash equilibrium → that **may not stabilize!**

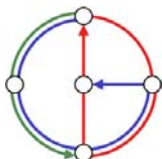


# Stability for general $\alpha$ ?

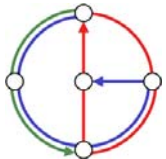
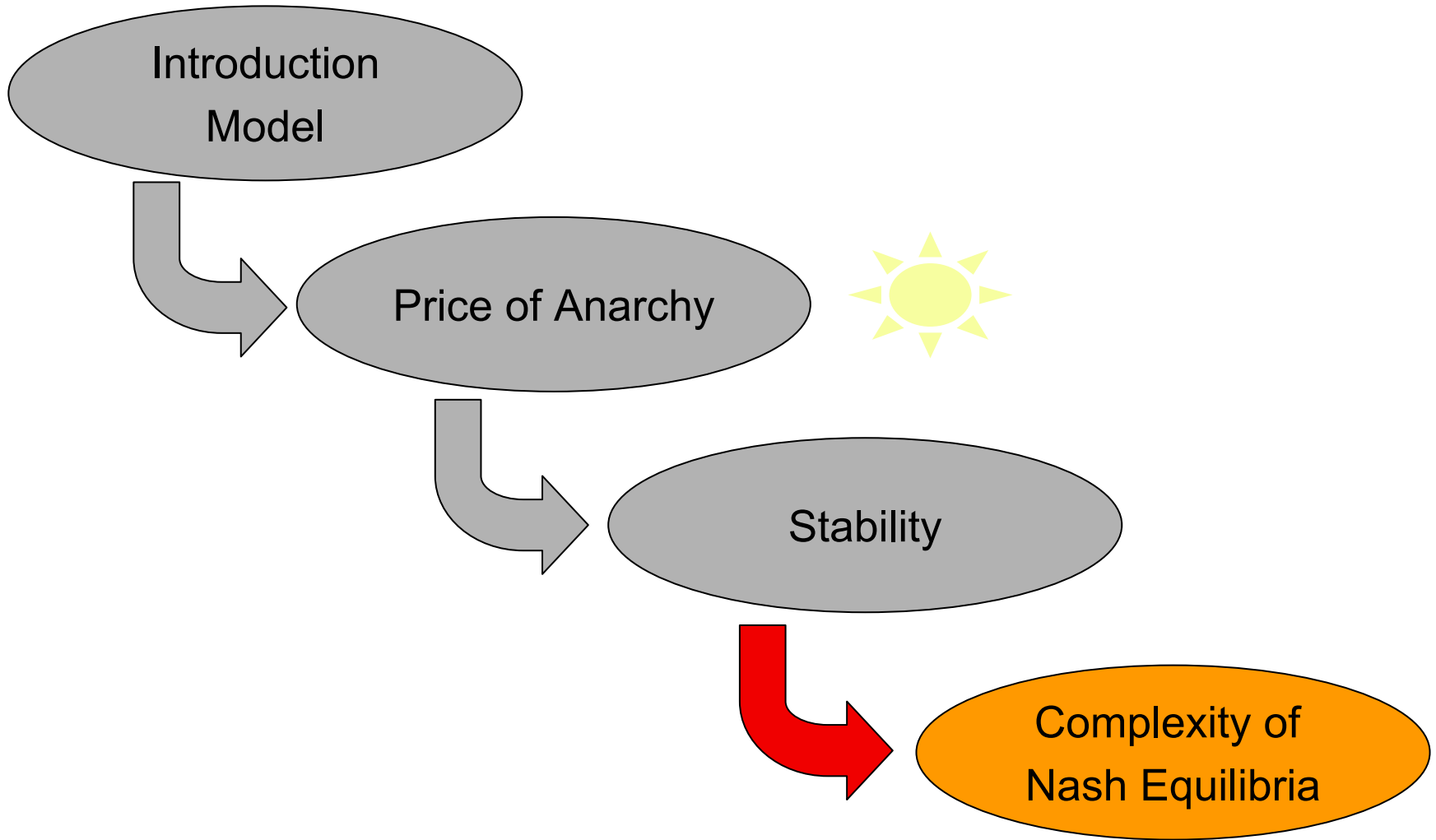
- So far, only a result for  $\alpha=0.6$
- With a trick, we can generalize it to **all magnitudes of  $\alpha$**
- Idea, replace one peer by a **cluster of peers**
- Each **cluster has  $k$  peers**  $\rightarrow$  The network is instable for  $\alpha=0.6k$
- Trick: between clusters, at most **one link** is formed (**larger  $\alpha \rightarrow$  larger groups**); this link then changes continuously as in the case of  $k=1$ .



$\delta \dots$  arbitrary small number



# Overview



# Complexity issues...

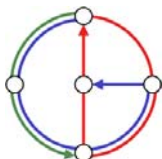


- **Selfishness can cause instability!**  
(even in the absence of churn, mobility, dynamism....)
- Can we (at least) **determine** whether a given **P2P network is stable?**  
(assuming that there is no churn, etc...)

→ What is the **complexity of stability**...???



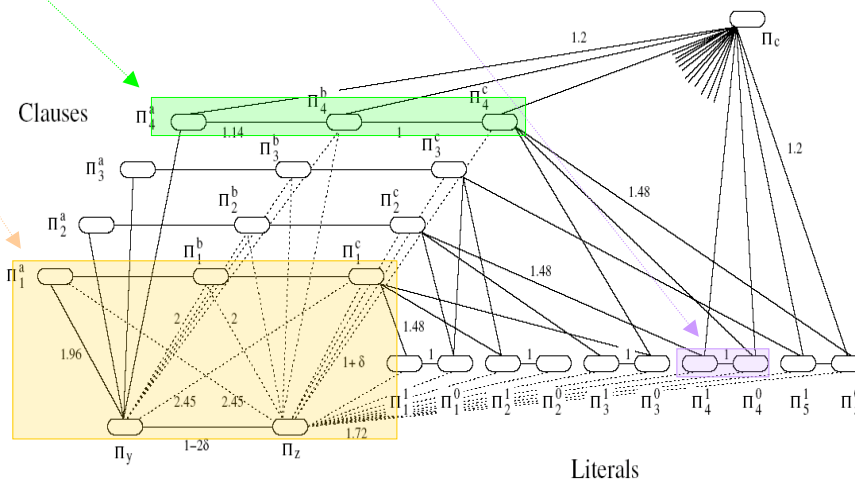
**Determining whether a  
P2P network has a (pure)  
Nash equilibrium is NP-hard!**





# Complexity of Nash Equilibrium

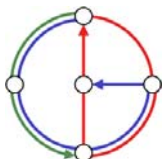
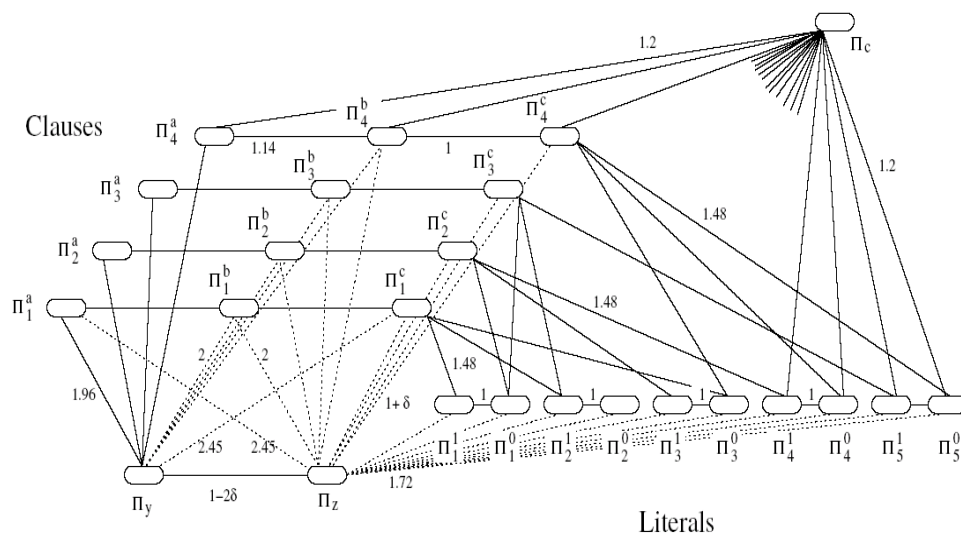
- Idea: **Reduction from 3-SAT** in CNF form (each clause has 3 literals)
  - Proof idea: Polynomial time reduction: SAT formula  $\rightarrow$  distribution of nodes in metric space
  - If **each clause** is satisfiable  $\rightarrow$  there exists a Nash equilibrium
  - Otherwise, it does not.
  - As reduction is fast, determining the complexity **must also be NP-hard**, like 3-SAT!
  - (Remark: Special 3-SAT, each variable in at most 3 clauses, still NP hard.)
- Arrange nodes as below
  - For each clause, **our old instable network!** (cliques  $\rightarrow$  for all magnitudes of  $\alpha$ )
  - Distances not shown are given by **shortest path metric**
  - **Not Euclidean metric** anymore, but **triangle inequality** etc. ok!
  - Two clusters at bottom, **three clusters per clause**, plus a **cluster for each literal** (positive and negative variable)
  - Clause **cluster node on the right has short distance** to those literal clusters appearing in the clause!



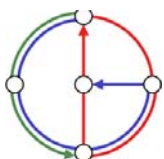
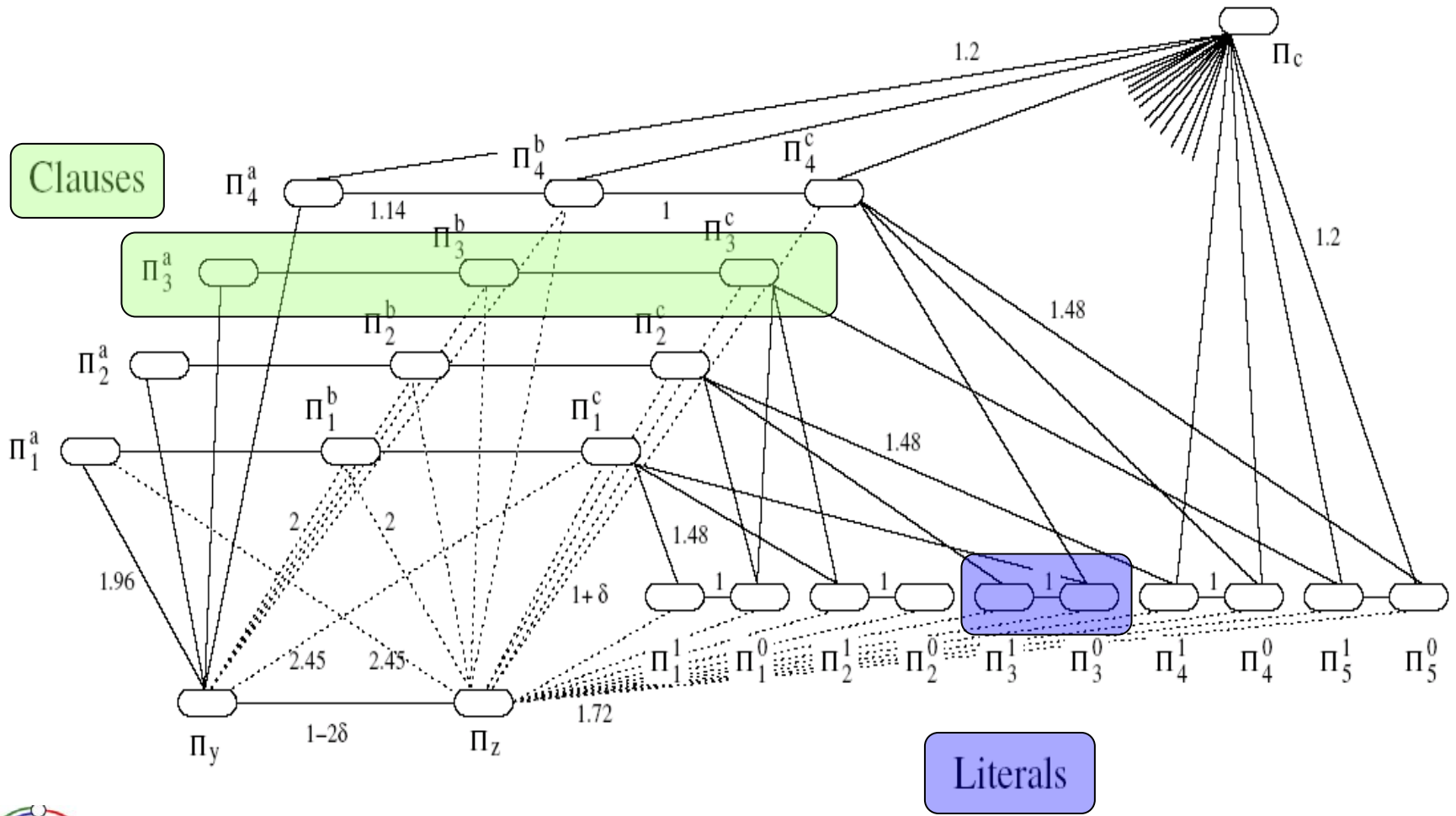
Stefan Schmid @

# Complexity of Nash Equilibrium

- Main idea: The **literal clusters help to stabilize!**
  - **short distance from  $\Pi^c$  (by construction), and maybe from  $\Pi_z$**
- The clue:  $\Pi_z$  can only **connect to one literal** per variable! (“assignment”)
  - Gives the satisfiable assignment making all clauses stable.
- If a clause has **only unsatisfied literals**, the paths become too large and the corresponding clause becomes instable!
  - Otherwise the network is stable, i.e., there exists a Nash equilibrium.

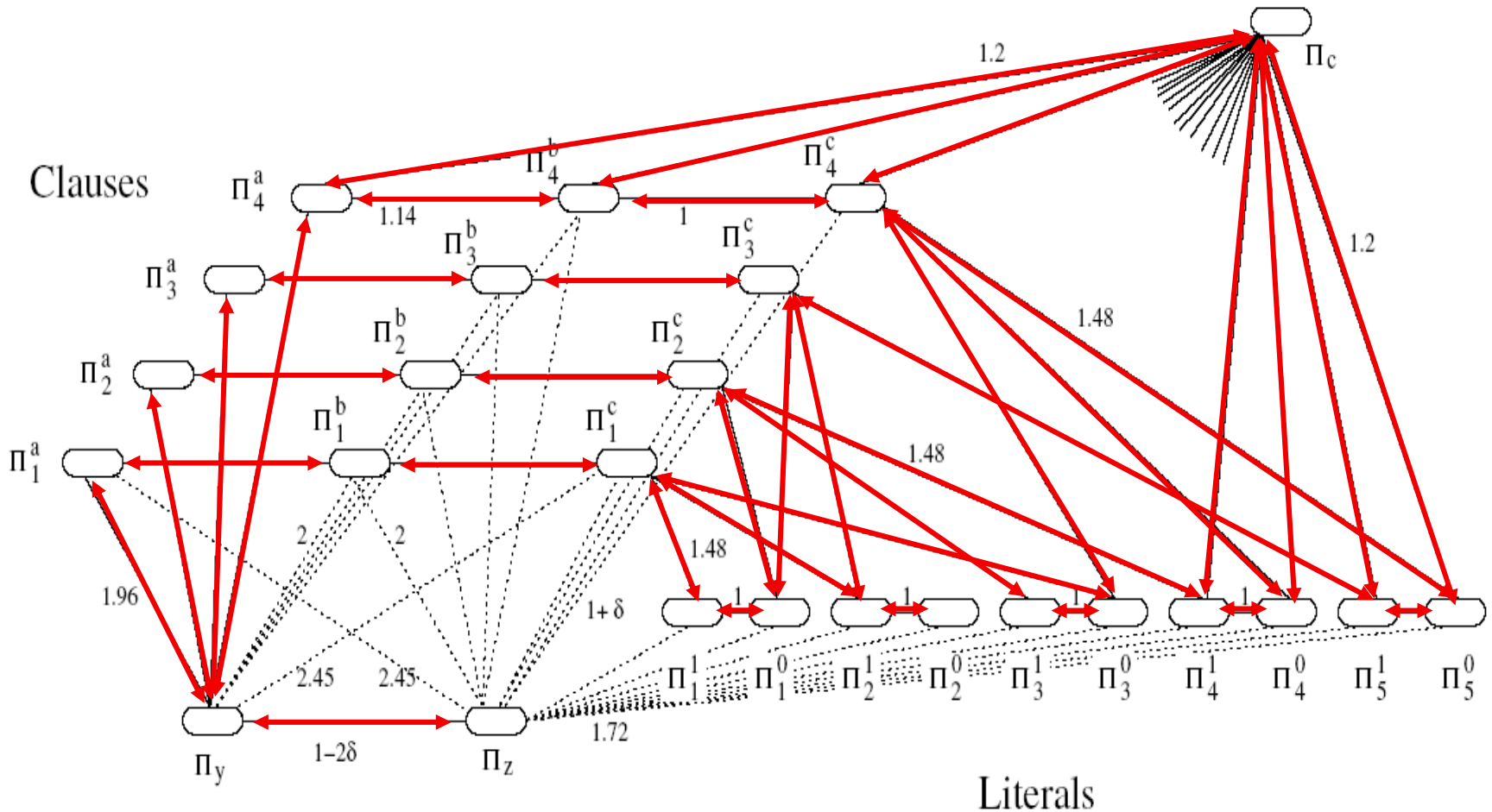


# Complexity of Nash Equilibrium



# Complexity of Nash Equilibrium

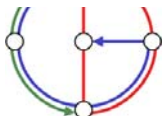
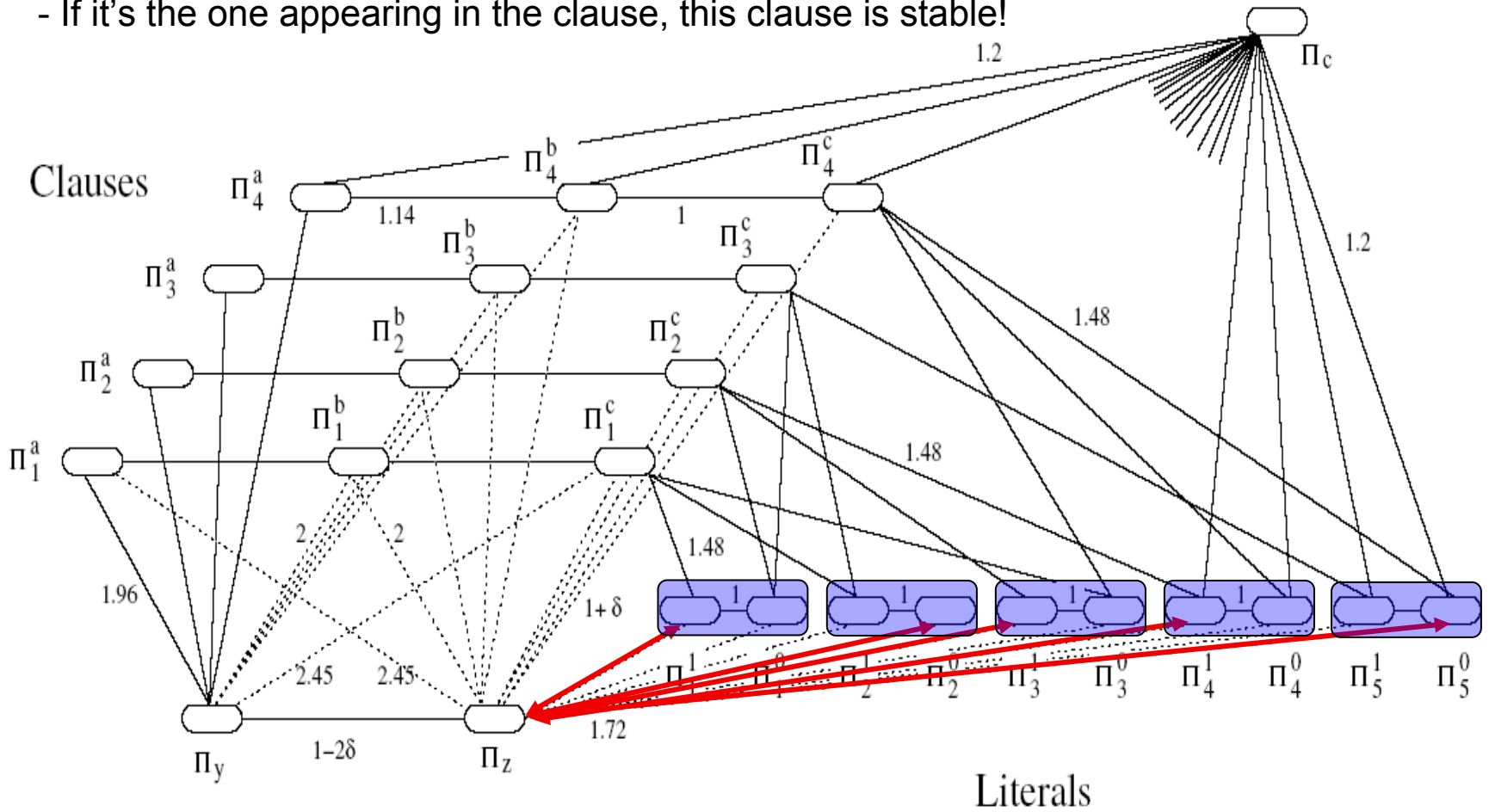
- It can be shown: In any Nash equilibrium, these links must exist...



# Complexity of Nash Equilibrium

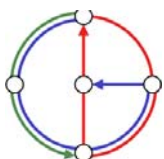
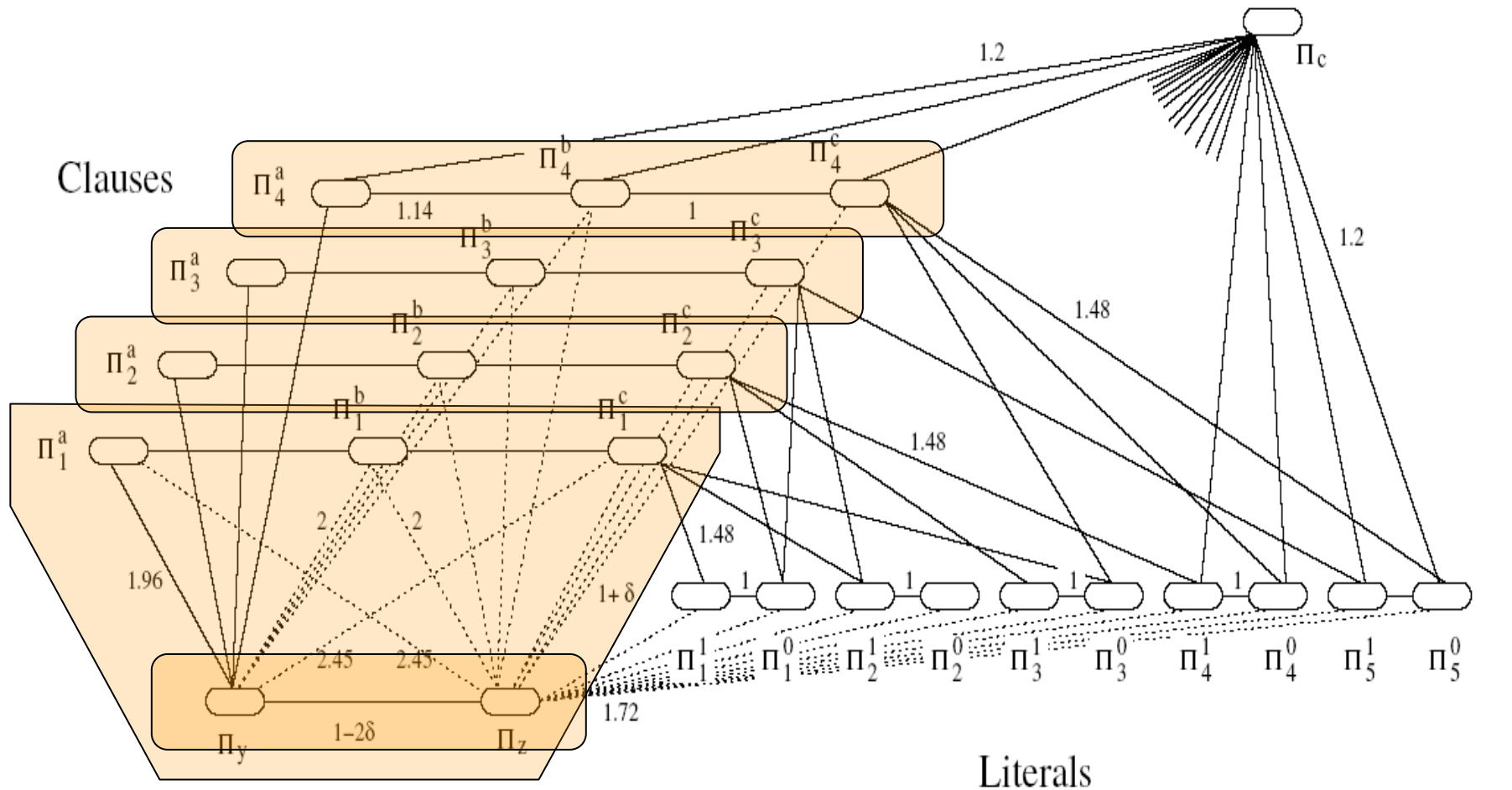
Special 3-SAT: Each variable in at most 3 clauses!

- Additionally,  $\Pi_z$  has exactly **one link to one literal** of each variable!
  - Defines the **"assignment"** of the variables for the formula.
  - If it's the one appearing in the clause, this clause is stable!



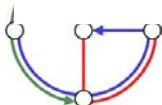
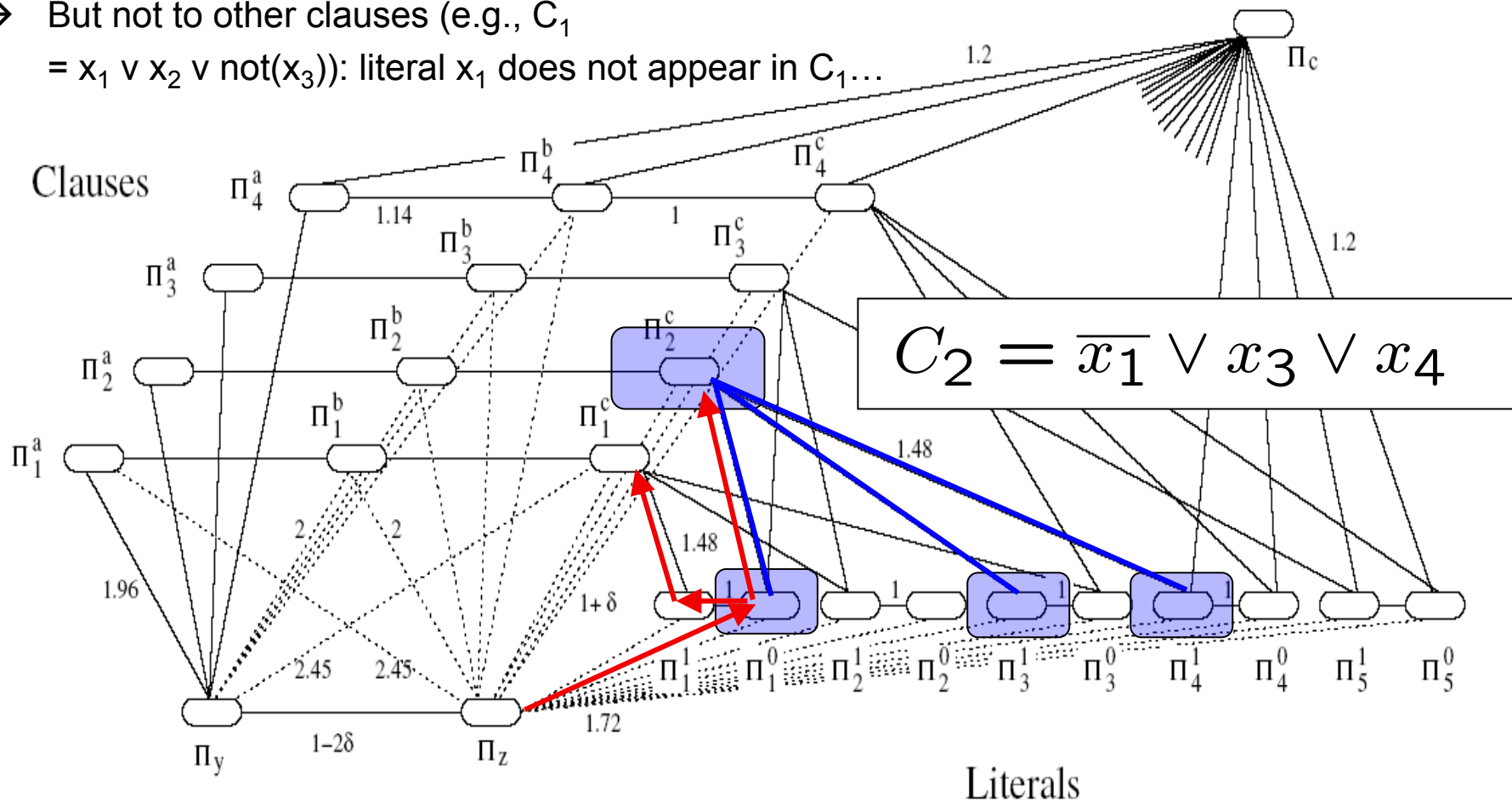
# Complexity of Nash Equilibrium

- Such a subgraph  $(\Pi_y, \Pi_z, \text{Clause})$  does not converge by itself...



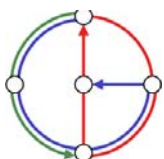
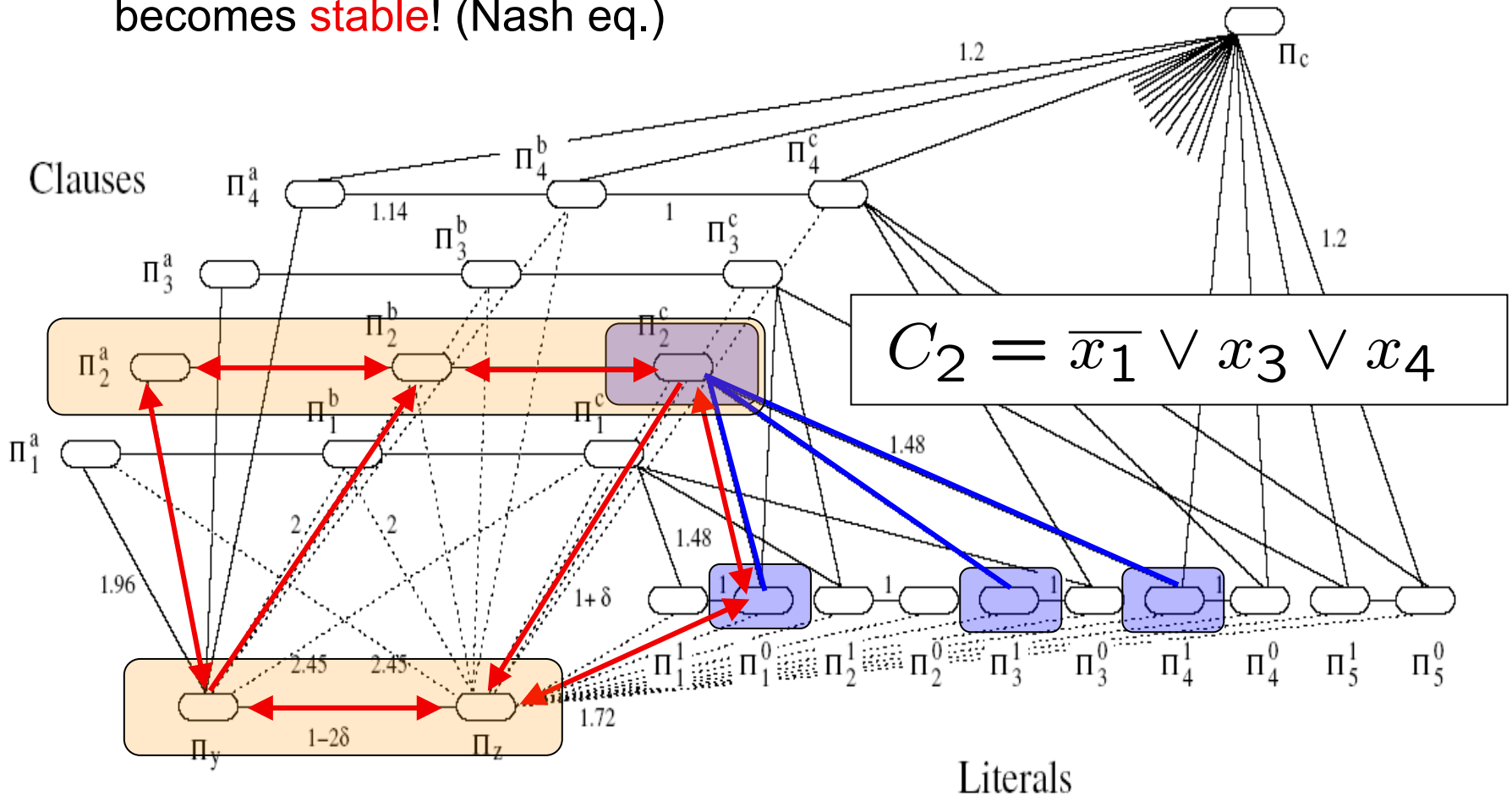
# Complexity of Nash Equilibrium

- In NE, each node-set  $\Pi^c$  is **connected** to those literals that are in the clause (not to other!)
  - if  $\Pi_z$  has link to  $\text{not}(x_1)$ , there is a “**short-cut**” to such clause-nodes, and  $C_2$  is **stable**
  - But not to other clauses (e.g.,  $C_1 = x_1 \vee x_2 \vee \text{not}(x_3)$ ): literal  $x_1$  does not appear in  $C_1 \dots$



# Complexity of Nash Equilibrium

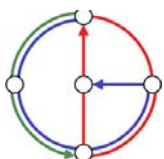
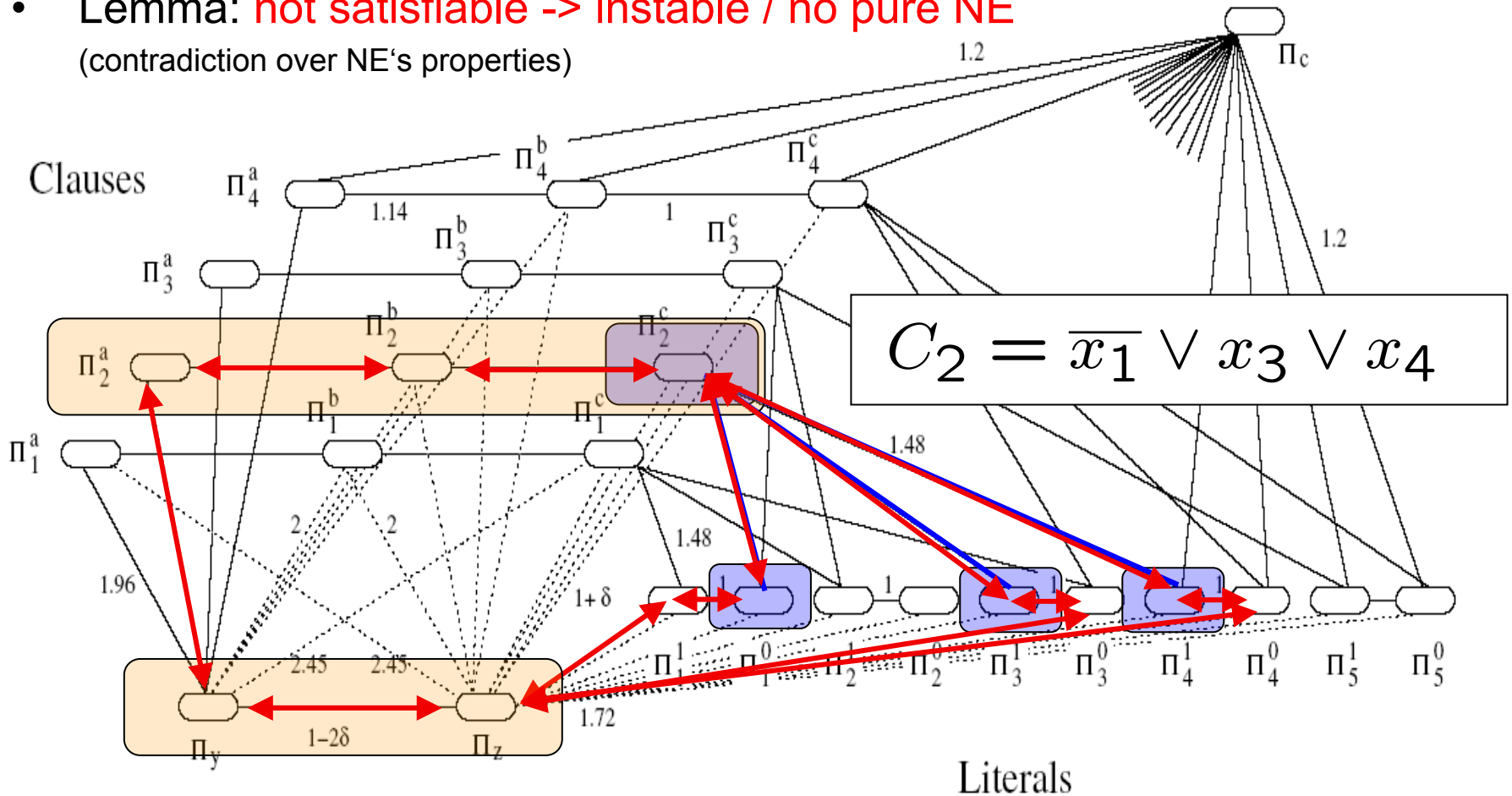
- A clause to which  $\Pi_z$  has a “short-cut” via a literal in this clause becomes **stable!** (Nash eq.)





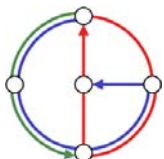
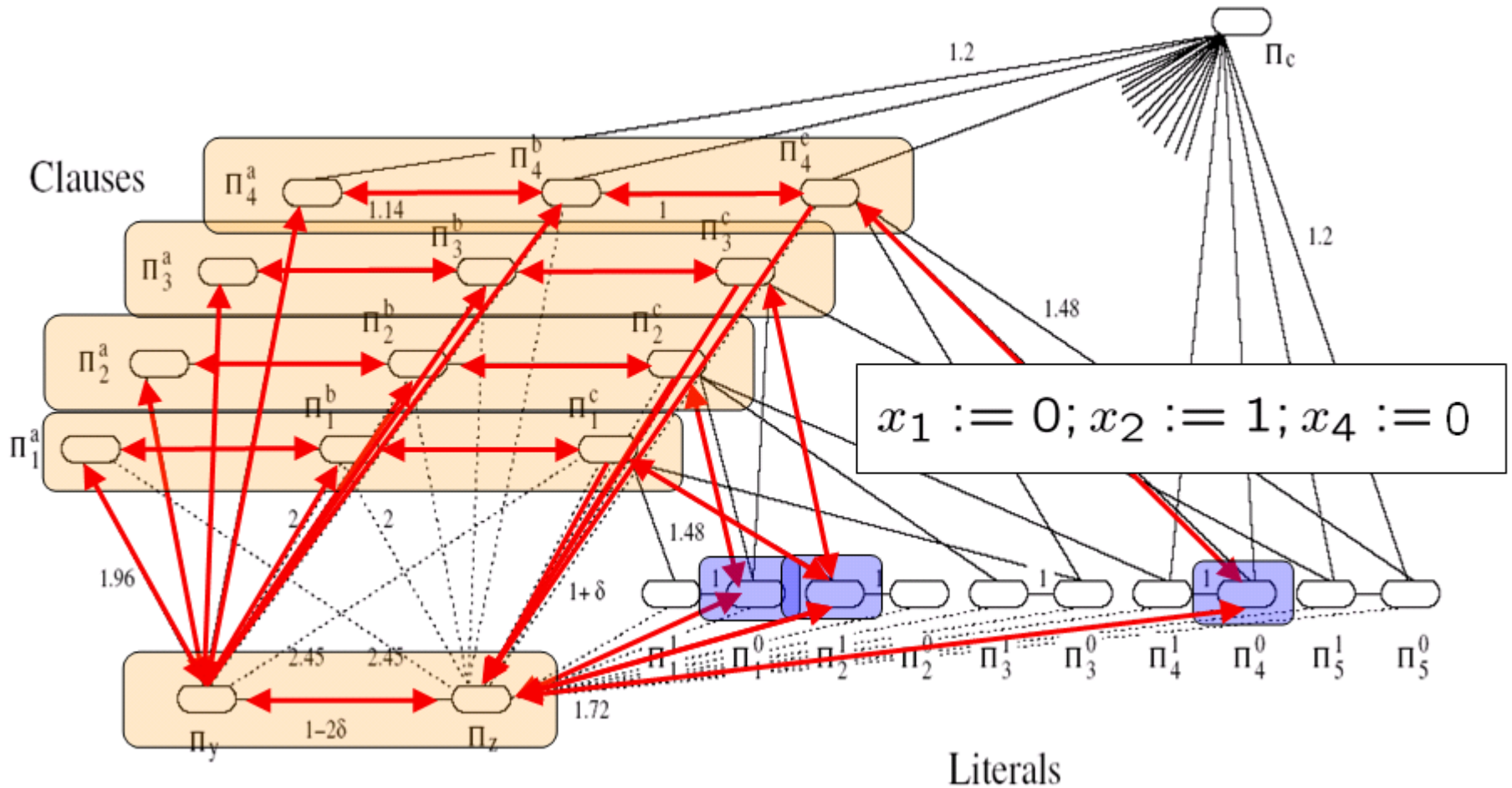
# Complexity of Nash Equilibrium

- If there is **no** such “short-cut” to a clause, the clause remains **instable!**
- Lemma: **not satisfiable** -> **instable** / **no pure NE**  
(contradiction over NE's properties)



# Complexity of Nash Equilibrium

- Example: satisfiable assignment  $\rightarrow$  all clauses stable  $\rightarrow$  pure NE



# The Topologies formed by Selfish Peers

- Selfish **neighbor selection** in **unstructured P2P systems**
- Goals of selfish peer:
  - (1) Maintain links only to a few neighbors (small **out-degree**)
  - (2) Small **latencies** to all other peers in the system (fast lookups)

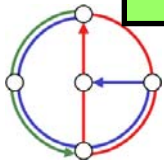


What is the impact on the P2P topologies?

**Price of Anarchy  $\in \Theta(\min\{\alpha, n\})$**

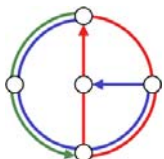
**Determining whether a P2P network has a (pure) Nash equilibrium is NP-hard!**

**Even in the absence of churn, mobility or other sources of dynamism, the system may never stabilize**



# Future Directions – Open Problems

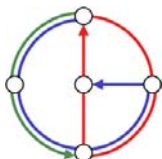
- Nash equilibrium **assumes full knowledge** about topology!
  - this is of course unrealistic
  - incorporate aspects of **local knowledge** into model
- Current model does not consider **routing** or **congestion** aspects!
  - also, why should every node be connected to every other node?  
(i.e., **infinite costs if not**? Not appropriate in Gnutella or so!)
- **Mechanism design**: How to guarantee stability/efficiency..?
- More **practical**: what is the parameter  $\alpha$  in real P2P networks?
- Lots more:
  - Algorithms to compute **social opt** of locality game?
  - Quality of **mixed** Nash equilibria?
  - Is it also hard to determine complexity for **Euclidean metrics**?
  - Computation of other equilibria
  - Comparisons to unilateral and bilateral games, and explanations?



# Conclusion



- Peer-to-peer computing continues posing exciting research questions!
- **Dynamics:**
  - Measurements in **practice**? BitTorrent vs Skype vs Joost?
  - What are good models? Worst-case churn or Poisson model? Max-min algebra?
  - Relaxed requirements? Simulated topology may break, but eventually **self-stabilize**?
  - Other forms of dynamics besides node churn? Dynamic bandwidth?
- **Non-cooperation:**
  - Game-theoretic assumptions often unrealistic, e.g., **complete knowledge** of system's state (e.g., Nash equilibrium, or knowledge of all shortest paths)
  - Algorithmic **mechanism design**: How to cope with different forms of selfishness? Incentives to establish „good links“?
  - **Social questions**: Why are so many anonymous participants still sharing their resources?



Thank you.



**Thank you for your interest.**

**All presented papers can be found at:  
<http://dgc.ethz.ch/members/stefan.html>**

